



## Scaling parameter

Parameter name	Parameter description	Chosen parameter values
$k, p, b$	<p><math>k</math> – number of rows and columns (square grid consists of <math>k \times k</math> DCE nodes and <math>4 \cdot k</math> DTE nodes attached on the borders of a square);</p> <p><math>p</math> – number of packets in each section of internal buffer;</p> <p><math>b</math> – available size of internal buffer.</p> <p><math>p</math> and <math>b</math> define initial marking and do not affect the model structure.</p>	$k = 2, 4, 8, 10, 13$ , with $p = k/2$ and $b = k$

## Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
$k$	$P = 13 \cdot k^2 + 8 \cdot k$	$T = 16 \cdot k^2 + 4 \cdot k$	$A = 64 \cdot k^2 + 16 \cdot k$
$k = 2$	68	72	288
$k = 4$	240	272	1088
$k = 8$	896	1056	4224
$k = 10$	1380	1640	6560
$k = 13$	2301	2756	11024

## Structural properties

<b>ordinary</b> — all arcs have multiplicity one .....	✓
<b>simple free choice</b> — all transitions sharing a common input place have no other input place .....	✗ (a)
<b>extended free choice</b> — all transitions sharing a common input place have the same input places .....	✗ (b)
<b>state machine</b> — every transition has exactly one input place and exactly one output place .....	✗ (c)
<b>marked graph</b> — every place has exactly one input transition and exactly one output transition .....	✗ (d)
<b>connected</b> — there is an undirected path between every two nodes (places or transitions) .....	✓ (e)
<b>strongly connected</b> — there is a directed path between every two nodes (places or transitions) .....	✓ (f)
<b>source place(s)</b> — one or more places have no input transitions .....	✗ (g)
<b>sink place(s)</b> — one or more places have no output transitions .....	✗ (h)
<b>source transition(s)</b> — one or more transitions have no input places .....	✗ (i)
<b>sink transitions(s)</b> — one or more transitions have no output places .....	✗ (j)
<b>loop-free</b> — no transition has an input place that is also an output place .....	✓ (k)
<b>conservative</b> — for each transition, the number of input arcs equals the number of output arcs .....	✓ (l)
<b>subconservative</b> — for each transition, the number of input arcs equals or exceeds the number of output arcs .....	✓ (m)
<b>nested units</b> — places are structured into hierarchically nested sequential units <sup>(n)</sup> .....	✗

(a) stated by [CÆSAR.BDD](#) version 2.2 on all 5 instances ( $k = 2, 4, 8, 10, 13$ ).

(b) stated by [CÆSAR.BDD](#) version 2.6 on all 5 instances ( $k = 2, 4, 8, 10, 13$ ).

(c) stated by [CÆSAR.BDD](#) version 2.2 on all 5 instances ( $k = 2, 4, 8, 10, 13$ ).

(d) stated by [CÆSAR.BDD](#) version 2.2 on all 5 instances ( $k = 2, 4, 8, 10, 13$ ).

(e) stated by [CÆSAR.BDD](#) version 2.2 on all 5 instances ( $k = 2, 4, 8, 10, 13$ ).

(f) stated by [CÆSAR.BDD](#) version 2.2 on all 5 instances ( $k = 2, 4, 8, 10, 13$ ).

(g) stated by [CÆSAR.BDD](#) version 2.2 on all 5 instances ( $k = 2, 4, 8, 10, 13$ ).

(h) stated by [CÆSAR.BDD](#) version 2.2 on all 5 instances ( $k = 2, 4, 8, 10, 13$ ).

(i) stated by [CÆSAR.BDD](#) version 2.2 on all 5 instances ( $k = 2, 4, 8, 10, 13$ ).

(j) stated by [CÆSAR.BDD](#) version 2.2 on all 5 instances ( $k = 2, 4, 8, 10, 13$ ).

(k) stated by [CÆSAR.BDD](#) version 2.2 on all 5 instances ( $k = 2, 4, 8, 10, 13$ ).

(l) stated by [CÆSAR.BDD](#) version 2.2 on all 5 instances ( $k = 2, 4, 8, 10, 13$ ).

(m) stated by [CÆSAR.BDD](#) version 2.2 on all 5 instances ( $k = 2, 4, 8, 10, 13$ ).

(n) the definition of Nested-Unit Petri Nets (NUPN) is available from <http://mcc.lip6.fr/nupn.php>

## Behavioural properties

- safe** — *in every reachable marking, there is no more than one token on a place* ..... ✗ <sup>(o)</sup>  
**deadlock** — *there exists a reachable marking from which no transition can be fired* ..... ✓ <sup>(p)</sup>  
**reversible** — *from every reachable marking, there is a transition path going back to the initial marking* ..... ✗  
**quasi-live** — *for every transition  $t$ , there exists a reachable marking in which  $t$  can fire* ..... ✓  
**live** — *for every transition  $t$ , from every reachable marking, one can reach a marking in which  $t$  can fire* ..... ✗

## Size of the marking graphs

Parameter	Number of reach-able markings	Number of tran-sition firings	Max. number of tokens per place	Max. number of tokens per marking
$k = 2$	?	?	?	48 <sup>(q)</sup>
$k = 4$	?	?	?	272 <sup>(r)</sup>
$k = 8$	?	?	?	1824 <sup>(s)</sup>
$k = 10$	?	?	?	3440 <sup>(t)</sup>
$k = 13$	?	?	?	6981 <sup>(u)</sup>

## Other properties

Model is  $4 \cdot p + b$  bounded – the sum of tokens in DCE internal buffer places. Model is P/T-invariant for any natural  $k$  as proven in [1,2].

<sup>(o)</sup> stated by CÆSAR.BDD version 2.2 on all 5 instances ( $k = 2, 4, 8, 10, 13$ ).

<sup>(p)</sup> proven in [1,2]; checked by the Tina <http://www.laas.fr/tina> tool version 3.3.0 as well as other behavioural properties for small values of parameter  $k$ .

<sup>(q)</sup> number of initial tokens, because the net is conservative.

<sup>(r)</sup> number of initial tokens, because the net is conservative.

<sup>(s)</sup> number of initial tokens, because the net is conservative.

<sup>(t)</sup> number of initial tokens, because the net is conservative.

<sup>(u)</sup> number of initial tokens, because the net is conservative.