

*This form is a summary description of the model entitled “JoinFreeModules” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.*

## Description

Liveness and boundedness are well-known Petri net properties that are fundamental for many real-world applications, including embedded systems, and are hard to check. So as to alleviate this difficulty, particular subclasses are often considered. The weighted join-free class has a limited expressiveness, since it forbids synchronizations. However, even with this strong structural limitation, this class remains hard to analyze and appears as a fundamental module of more complex classes, such as weighted asymmetric-choice nets whose behavior is strongly related to its join-free modules [1].

The model provided is inspired from a paper of T. Hujsa and R. Devillers [1] which investigates the relationship between liveness and structural boundedness (meaning boundedness for every initial marking) in some subclasses of weighted Petri nets, notably join-free nets. The paper also studies the monotonicity of liveness, meaning its preservation upon any increase of the live marking considered.

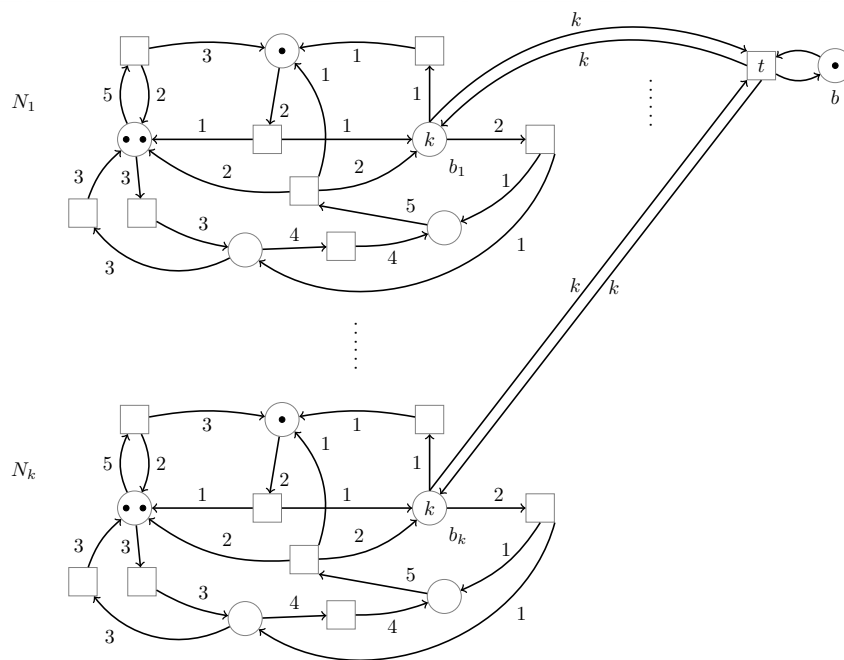
The model is based on one parameter  $k$ , which denotes:

- the number of join-free modules,
- the initial number of tokens in each buffer (place)  $b_1, \dots, b_k$
- the amount of tokens read and written (as a self-loop) through one firing of  $t$  (outside the modules) in each  $b_1, \dots, b_k$ .

Each module  $N_i$ ,  $i \in \{1, \dots, k\}$ , is a structurally live and bounded join-free net containing the buffer  $b_i$ . A single transition  $t$  checks the existence of  $i$  tokens in each buffer  $b_i$ . Each  $N_i$  can represent a sub-program with asynchronous iterations to be executed concurrently on different processors. Each new iteration in  $N_i$  reads some data items (tokens) in its buffer  $b_i$ .

The sub-program  $t$  reads  $k$  data items in each buffer  $b_i$ , computes a function on them and updates the buffer  $b$  with the result. The purpose of this operation is to analyse the current progress of each module and to gather this updated information in  $b$ . Due to this synchronisation  $t$ , the global system is not join-free.

Each join-free module  $N_i$  is live for  $k = 3$ , becomes non-live for  $k = 4, 5, 6$  (proving that liveness is not monotonic in the structurally bounded join-free class) and becomes live again for all  $k \geq 7$  (this stems partly from a result of [2]). Each  $N_i$  is live and reversible for all  $k \geq 8$ , implying that the global system is live and reversible for every  $k \geq 8$ , at least.”



For each  $i \in \{1, \dots, k\}$ , the subnet  $N_i$  contains a place  $b_i$  with  $k$  initial tokens. The transition  $t$  can fire when at least  $k$  tokens are present in each  $b_i$ .

## References

- [1] Thomas Hujsa and Raymond Devillers. *On Liveness and Deadlockability in Subclasses of Weighted Petri Nets*. Proceedings of the 38th International Conference on Application and Theory of Petri Nets and Concurrency (Petri Nets'17), 2017.
- [2] Jean-Marc Delosme, Thomas Hujsa, and Alix Munier-Kordon. *Polynomial sufficient conditions of well-behavedness for weighted Join-Free and Choice-Free systems*. In Proceedings of the 13th International Conference on Application of Concurrency to System Design (ACSD'13). pages 90–99, 2013.

## Scaling parameter

Parameter name	Parameter description	Chosen parameter values
$k$	The number of initial tokens in each buffer $b_i$ (in the module $N_i$ ), the number of tokens consumed and produced by each firing of $t$ in the same buffers, and the number of modules	3, 4, 5, 10, 20, 50, 100, 200, 500, 1000, 2000, 5000

## Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
$k$	$5k + 1$	$8k + 1$	$23k + 2$
$k = 3$	16	25	71
$k = 4$	21	33	94
$k = 5$	26	41	117
$k = 10$	51	81	232
$k = 20$	101	161	462
$k = 50$	251	401	1 152
$k = 100$	501	801	2 302
$k = 200$	1 001	1 601	4 602
$k = 500$	2 501	4 001	11 502
$k = 1000$	5 001	8 001	23 002
$k = 2000$	10 001	16 001	46 002
$k = 5000$	25 001	40 001	115 002

## Structural properties

<b>ordinary</b> — all arcs have multiplicity one .....	X
<b>simple free choice</b> — all transitions sharing a common input place have no other input place .....	X (a)
<b>extended free choice</b> — all transitions sharing a common input place have the same input places .....	X (b)
<b>state machine</b> — every transition has exactly one input place and exactly one output place .....	X (c)
<b>marked graph</b> — every place has exactly one input transition and exactly one output transition .....	X (d)
<b>connected</b> — there is an undirected path between every two nodes (places or transitions) .....	✓ (e)
<b>strongly connected</b> — there is a directed path between every two nodes (places or transitions) .....	✓ (f)
<b>source place(s)</b> — one or more places have no input transitions .....	X (g)
<b>sink place(s)</b> — one or more places have no output transitions .....	X (h)
<b>source transition(s)</b> — one or more transitions have no input places .....	X (i)
<b>sink transitions(s)</b> — one or more transitions have no output places .....	X (j)
<b>loop-free</b> — no transition has an input place that is also an output place .....	X (k)
<b>conservative</b> — for each transition, the number of input arcs equals the number of output arcs .....	X (l)
<b>subconservative</b> — for each transition, the number of input arcs equals or exceeds the number of output arcs .....	✓ (m)
<b>nested units</b> — places are structured into hierarchically nested sequential units <sup>(n)</sup> .....	X

## Behavioural properties

<b>safe</b> — in every reachable marking, there is no more than one token on a place .....	X (o)
<b>deadlock</b> — there exists a reachable marking from which no transition can be fired .....	?
<b>reversible</b> — from every reachable marking, there is a transition path going back to the initial marking .....	?
<b>quasi-live</b> — for every transition $t$ , there exists a reachable marking in which $t$ can fire .....	?

(a) the net is not ordinary.

(b) the net is not ordinary.

(c) the net is not ordinary.

(d) the net is not ordinary.

(e) stated by [CÆSAR.BDD](#) version 2.7 on all 12 instances (see all aforementioned parameter values).

(f) stated by [CÆSAR.BDD](#) version 2.7 on all 12 instances (see all aforementioned parameter values).

(g) stated by [CÆSAR.BDD](#) version 2.7 on all 12 instances (see all aforementioned parameter values).

(h) stated by [CÆSAR.BDD](#) version 2.7 on all 12 instances (see all aforementioned parameter values).

(i) stated by [CÆSAR.BDD](#) version 2.7 on all 12 instances (see all aforementioned parameter values).

(j) stated by [CÆSAR.BDD](#) version 2.7 on all 12 instances (see all aforementioned parameter values).

(k) stated by [CÆSAR.BDD](#) version 2.7 on all 12 instances (see all aforementioned parameter values).

(l) stated by [PNML2NUPN](#) 1.5.3 on all 12 instances (see all aforementioned parameter values).

(m) stated by [PNML2NUPN](#) 1.5.3 on all 12 instances (see all aforementioned parameter values).

(n) the definition of Nested-Unit Petri Nets (NUPN) is available from <http://mcc.lip6.fr/nupn.php>

(o) stated by [CÆSAR.BDD](#) version 2.7 on all 12 instances (see all aforementioned parameter values).

live — for every transition  $t$ , from every reachable marking, one can reach a marking in which  $t$  can fire .....

### Size of the marking graphs

Parameter	Number of reach-able markings	Number of tran-sition firings	Max. number of tokens per place	Max. number of tokens per marking
For any $k$ :	–	–	$\leq k + 3$	$k \cdot (k + 3) + 1$
$k = 3$	35 937	225 450	$\leq 6$	19 <sup>(p)</sup>
$k = 4$	14 776 336	138 230 321	$\leq 7$	29 <sup>(q)</sup>
$k = 5$	?	?	$\leq 8$	41 <sup>(r)</sup>
$k = 10$	?	?	$\leq 13$	131 <sup>(s)</sup>
$k = 20$	?	?	$\leq 23$	461 <sup>(t)</sup>
$k = 50$	?	?	$\leq 53$	2 651 <sup>(u)</sup>
$k = 100$	?	?	$\leq 103$	10 301 <sup>(v)</sup>
$k = 200$	?	?	$\leq 203$	40 601 <sup>(w)</sup>
$k = 500$	?	?	$\leq 503$	251 501 <sup>(x)</sup>
$k = 1000$	?	?	$\leq 1\,003$	1 003 001 <sup>(y)</sup>
$k = 2000$	?	?	$\leq 2\,003$	4 006 001 <sup>(z)</sup>
$k = 5000$	?	?	$\leq 5\,003$	2 5015 001 <sup>(aa)</sup>

### Other properties

P — and T – invariants exist and cover the net. Denoting by  $L$  the liveness property and by  $R$  the reversibility property, there exist markings satisfying:

- $L$  and  $R$
- $L$  and not  $R$
- not  $L$  and  $R$
- not  $L$  and not  $R$

For the value  $k = 4$ , a deadlock is reachable.

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<sup>(p)</sup> number of initial tokens, because the net is sub-conservative.  
<sup>(q)</sup> number of initial tokens, because the net is sub-conservative.  
<sup>(r)</sup> number of initial tokens, because the net is sub-conservative.  
<sup>(s)</sup> number of initial tokens, because the net is sub-conservative.  
<sup>(t)</sup> number of initial tokens, because the net is sub-conservative.  
<sup>(u)</sup> number of initial tokens, because the net is sub-conservative.  
<sup>(v)</sup> number of initial tokens, because the net is sub-conservative.  
<sup>(w)</sup> number of initial tokens, because the net is sub-conservative.  
<sup>(x)</sup> number of initial tokens, because the net is sub-conservative.  
<sup>(y)</sup> number of initial tokens, because the net is sub-conservative.  
<sup>(z)</sup> number of initial tokens, because the net is sub-conservative.  
<sup>(aa)</sup> number of initial tokens, because the net is sub-conservative.