

*This form is a summary description of the model entitled “HypercubeCommunicationGrid” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.*

## Description

Hypercube communication grid model [1,2] is composed of nodes which represent data communication equipment (DCE) implementing packet forwarding based on store-and-forward principle. Each DCE has ports, situated on facets of a unit size hypercube, which work in full-duplex mode. Data terminal equipment (DTE) is attached on the hypercube borders. Each DTE receives and sends packets.

Remind that, a  $d$ -dimension hypercube has  $2 \cdot d$  facets each represents a  $(d - 1)$ -dimension hypercube.

DCE index  $(i_1, i_2, \dots, i_d)$ , where  $1 \leq i_j \leq k$ ,  $1 \leq j \leq d$ , reflects its location within hypercube. Port index  $(r, n)$  consists of dimension number  $1 \leq r \leq d$ , a facet is perpendicular to, and direction number  $1 \leq n \leq 2$ , where  $n = 1$  represents the direction to the origin of coordinates and  $n = 2$  represents the direction to infinity.

DCE model contains an internal buffer represented with  $2 \cdot d + 1$  places: the available buffer size and buffer sections for storing packets forwarded to the corresponding ports.

Each of  $2 \cdot d$  DCE ports has two tracts: input and output. Memory of a tract is represented with two places – the tract buffer and the tract buffer available capacity (usually equal to unit). An output tract work is modeled by a single transition taking a packet from the corresponding section of the internal buffer and putting it into the tract buffer. An input tract work is modeled by  $2 \cdot d - 1$  transitions forwarding arrived packet from the tract buffer to the corresponding section of the internal buffer except of the arrival port number.

A hypercube is composed via merging tract places of neighbor DCE which has a common facet: input tract of one DCE with output tract of the other DCE and vice versa.

On the borders, which constitute  $2 \cdot d$  hypercubes of dimension  $d - 1$ , DTE models are attached. A simple DTE model is represented with a single transition that receives a packet from a neighbor DCE output tract and sends a packet into the neighbor DCE input tract.

For planar case when  $d = 2$ , models are described in [1,3] with simplified notation of ports.

## References

- [1] Zaitsev D.A., Zaitsev I.D., Shmeleva T.R. Infinite Petri Nets as Models of Grids (pp. 187-204). Chapter 19 in Mehdi Khosrow-Pour (Ed.) Encyclopedia of Information Science and Technology, Third Edition (10 Volumes). IGI-Global: USA, 2014.
- [2] Zaitsev D.A., Shmeleva T.R. Hypercube communication structures analysis via parametric Petri nets. Proceedings of 24th UK Performance Engineering Workshop (UKPEW 2008), 3-4 July 2008, Department of Computing, Imperial College London, p. 358-371.
- [3] Shmeleva T.R., Zaitsev D.A., Zaitsev I.D. Analysis of Square Communication Grids via Infinite Petri Nets. Transactions of Odessa National Academy of Telecommunication, no. 1, 2009, p. 27-35.
- [4] A C program that generates  $k^d$  hypercube can be downloaded from <http://daze.ho.ua/tinaz.zip>

## Scaling parameter

Parameter name	Parameter description	Chosen parameter values
$d, k, p, b$	$d$ is the number of dimensions; $k$ is the hypercube size of $k^d$ DCE nodes and $2 \cdot d \cdot k^{d-1}$ DTE nodes; $p$ is the number of packets in each section of internal buffer; $b$ is the available size of internal buffer; $p$ and $b$ define the initial marking and do not affect the model structure.	$(d, k) = (3, 4), (4, 3), (5, 3)$ with $p = k$ and $b = d \cdot k$

## Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
$(d, k)$	$P = 6 \cdot d \cdot k^d + k^d + 4 \cdot d \cdot k^{d-1}$	$T = 4 \cdot d^2 \cdot k^d + 2 \cdot d \cdot k^{d-1}$	$A = 16 \cdot d^2 \cdot k^d + 8 \cdot d \cdot k^{d-1}$
$(d = 3, k = 4)$	1408	2400	9600
$(d = 4, k = 3)$	2457	5400	21600
$(d = 5, k = 3)$	9153	25110	100440

## Structural properties

- ordinary — all arcs have multiplicity one ..... ✓
- simple free choice — all transitions sharing a common input place have no other input place ..... ✗ (a)
- extended free choice — all transitions sharing a common input place have the same input places ..... ✗ (b)
- state machine — every transition has exactly one input place and exactly one output place ..... ✗ (c)
- marked graph — every place has exactly one input transition and exactly one output transition ..... ✗ (d)
- connected — there is an undirected path between every two nodes (places or transitions) ..... ✓ (e)
- strongly connected — there is a directed path between every two nodes (places or transitions) ..... ✓ (f)
- source place(s) — one or more places have no input transitions ..... ✗ (g)
- sink place(s) — one or more places have no output transitions ..... ✗ (h)
- source transition(s) — one or more transitions have no input places ..... ✗ (i)
- sink transitions(s) — one or more transitions have no output places ..... ✗ (j)
- loop-free — no transition has an input place that is also an output place ..... ✓ (k)
- conservative — for each transition, the number of input arcs equals the number of output arcs ..... ✓ (l)
- subconservative — for each transition, the number of input arcs equals or exceeds the number of output arcs ..... ✓ (m)
- nested units — places are structured into hierarchically nested sequential units<sup>(n)</sup> ..... ✗

(a) stated by [CÆSAR.BDD](#) version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).

(b) stated by [CÆSAR.BDD](#) version 2.6 on all 3 instances ((3, 4), (4, 3), (5, 3)).

(c) stated by [CÆSAR.BDD](#) version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).

(d) stated by [CÆSAR.BDD](#) version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).

(e) stated by [CÆSAR.BDD](#) version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).

(f) stated by [CÆSAR.BDD](#) version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).

(g) stated by [CÆSAR.BDD](#) version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).

(h) stated by [CÆSAR.BDD](#) version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).

(i) stated by [CÆSAR.BDD](#) version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).

(j) stated by [CÆSAR.BDD](#) version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).

(k) stated by [CÆSAR.BDD](#) version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).

(l) stated by [CÆSAR.BDD](#) version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).

(m) stated by [CÆSAR.BDD](#) version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).

(n) the definition of Nested-Unit Petri Nets (NUPN) is available from <http://mcc.lip6.fr/nupn.php>

## Behavioural properties

- safe** — *in every reachable marking, there is no more than one token on a place* ..... ✗<sup>(o)</sup>  
**deadlock** — *there exists a reachable marking from which no transition can be fired* ..... ✗<sup>(p)</sup>  
**reversible** — *from every reachable marking, there is a transition path going back to the initial marking* ..... ✗  
**quasi-live** — *for every transition  $t$ , there exists a reachable marking in which  $t$  can fire* ..... ✓  
**live** — *for every transition  $t$ , from every reachable marking, one can reach a marking in which  $t$  can fire* ..... ✗

## Size of the marking graphs

Parameter	Number of reach-able markings	Number of tran-sition firings	Max. number of tokens per place	Max. number of tokens per marking
$(d = 3, k = 4)$	?	?	?	2784 <sup>(q)</sup>
$(d = 4, k = 3)$	?	?	?	3780 <sup>(r)</sup>
$(d = 5, k = 3)$	?	?	?	14175 <sup>(s)</sup>

## Other properties

Model is  $2 \cdot d \cdot p + b$  bounded — the sum of tokens in DCE internal buffer places. Model is P/T-invariant for any natural  $k$  as proven in [1,2]

<sup>(o)</sup> stated by C/ESAR.BDD version 2.2 on all 3 instances  $((3, 4), (4, 3), (5, 3))$ .

<sup>(p)</sup> proven in [1,2]; checked by the Tina <http://www.laas.fr/tina> tool version 3.3.0 as well as other behavioural properties for small values of parameters  $d, k$ .

<sup>(q)</sup> number of initial tokens, because the net is conservative.

<sup>(r)</sup> number of initial tokens, because the net is conservative.

<sup>(s)</sup> number of initial tokens, because the net is conservative.