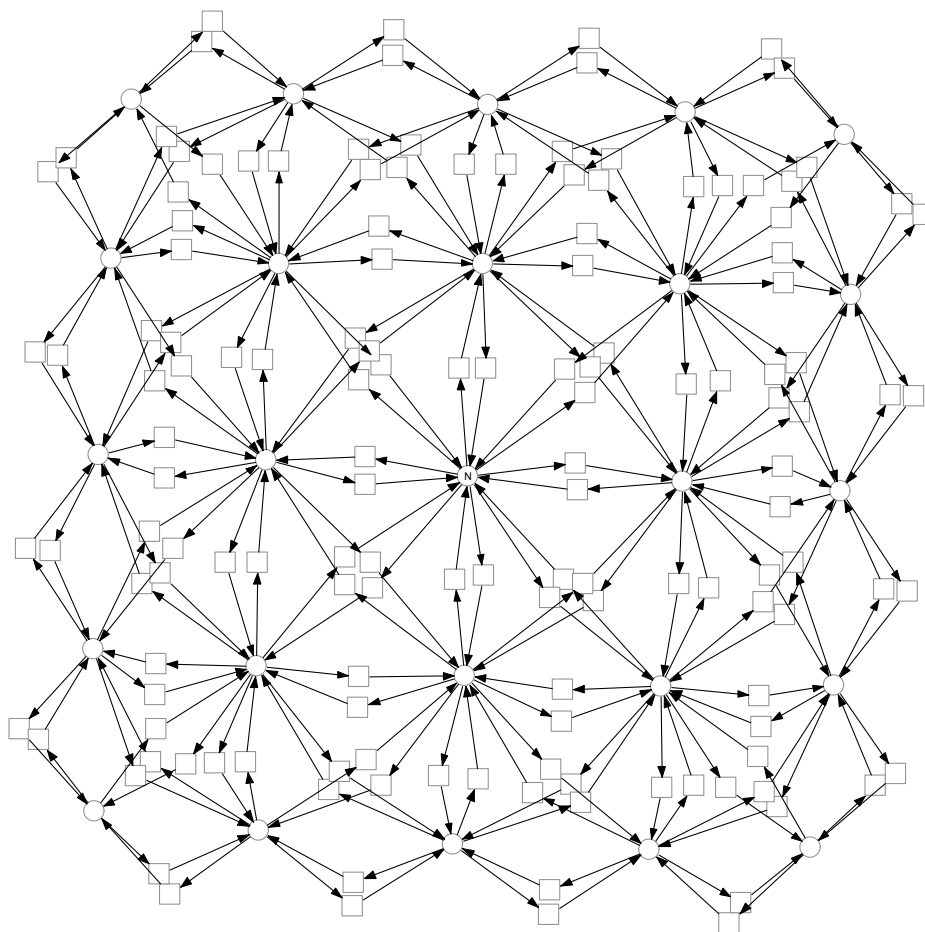


This form is a summary description of the model entitled “Diffusion2D” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.

Description

Diffusion in space is a basic process underlying many spatial (bio-) chemical processes, however typically considered either in the stochastic or continuous setting. The Petri net given here comes from [GHLS13], where it has been used to illustrate the generic modelling of space by use of coloured Petri nets. We discretise the space by a $D \times D$ rectangular grid, D being a model parameter, and deploy the 8-neighbourhood relation with reflecting boundary condition. The process starts with N tokens in the centre position. This model is easily scalable with well-known size of the model growth and its state space.



Graphical representation for $D = 5$, generated by unfolding and automatic layout of a coloured Petri net using Snoopy. There are N tokens in the centre position. All node names are set to invisible.

References

GHLS13 D. Gilbert, M. Heiner, F. Liu and N. Saunders: Colouring Space - A Coloured Framework for Spatial Modelling in Systems Biology; In Proc. PETRI NETS 2013, Milano, Springer, LNCS, volume 7927, pages 230–249, June 2013.

Scaling parameter

Parameter name	Parameter description	Chosen parameter values
D, N	grid size; i.e. there are $D \times D$ grid positions, and initially N tokens in the centre position.	$\langle D = 5, N = 10 \rangle, \langle D = 5, N = 50 \rangle,$ $\langle D = 5, N = 100 \rangle, \langle D = 5, N = 150 \rangle,$ $\langle D = 5, N = 200 \rangle, \langle D = 5, N = 250 \rangle,$ $\langle D = 5, N = 300 \rangle, \langle D = 5, N = 350 \rangle,$ $\langle D = 10, N = 10 \rangle, \langle D = 10, N = 50 \rangle,$ $\langle D = 10, N = 100 \rangle, \langle D = 10, N = 150 \rangle,$ $\langle D = 10, N = 200 \rangle, \langle D = 20, N = 10 \rangle,$ $\langle D = 20, N = 50 \rangle, \langle D = 20, N = 100 \rangle,$ $\langle D = 20, N = 150 \rangle, \langle D = 30, N = 10 \rangle,$ $\langle D = 30, N = 50 \rangle, \langle D = 30, N = 100 \rangle,$ $\langle D = 30, N = 150 \rangle, \langle D = 40, N = 10 \rangle,$ $\langle D = 40, N = 50 \rangle, \langle D = 40, N = 100 \rangle,$ $\langle D = 40, N = 150 \rangle, \langle D = 50, N = 10 \rangle,$ $\langle D = 50, N = 50 \rangle, \langle D = 50, N = 100 \rangle,$ $\langle D = 50, N = 150 \rangle$

Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
D	D^2	$8D^2 - 12D + 4$	$2 \cdot T $
$\langle D = 5, N = \dots \rangle$	25	144	288
$\langle D = 10, N = \dots \rangle$	100	684	1368
$\langle D = 20, N = \dots \rangle$	400	2964	5928
$\langle D = 30, N = \dots \rangle$	900	6844	13688
$\langle D = 40, N = \dots \rangle$	1600	12324	24648
$\langle D = 50, N = \dots \rangle$	2500	19404	38808

Structural properties

- ordinary** — all arcs have multiplicity one ✓
- simple free choice** — all transitions sharing a common input place have no other input place ✓ (a)
- extended free choice** — all transitions sharing a common input place have the same input places ✓ (b)
- state machine** — every transition has exactly one input place and exactly one output place ✓ (c)
- marked graph** — every place has exactly one input transition and exactly one output transition ✗ (d)
- connected** — there is an undirected path between every two nodes (places or transitions) ✓ (e)
- strongly connected** — there is a directed path between every two nodes (places or transitions) ✓ (f)
- source place(s)** — one or more places have no input transitions ✗ (g)
- sink place(s)** — one or more places have no output transitions ✗ (h)
- source transition(s)** — one or more transitions have no input places ✗ (i)
- sink transitions(s)** — one or more transitions have no output places ✗ (j)
- loop-free** — no transition has an input place that is also an output place ✓ (k)
- conservative** — for each transition, the number of input arcs equals the number of output arcs ✓ (l)

(a) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
 (b) stated by [CÆSAR.BDD](#) version 2.6 on all 29 instances (see all aforementioned scaling parameter values).
 (c) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
 (d) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
 (e) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
 (f) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
 (g) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
 (h) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
 (i) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
 (j) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
 (k) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
 (l) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).

subconservative — for each transition, the number of input arcs equals or exceeds the number of output arcs ✓^(m)
nested units — places are structured into hierarchically nested sequential units⁽ⁿ⁾ ✗

Behavioural properties

safe — in every reachable marking, there is no more than one token on a place ✗^(o)
deadlock — there exists a reachable marking from which no transition can be fired ✗
reversible — from every reachable marking, there is a transition path going back to the initial marking ✓
quasi-live — for every transition t , there exists a reachable marking in which t can fire ✓^(p)
live — for every transition t , from every reachable marking, one can reach a marking in which t can fire ✓

Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
D, N	$\frac{(D^2+N-1)!}{(D^2-1)! \cdot N!}$?	N	N ^(q)
5, 10	1.3113E+8 ^(r)	5.5537E+9 ^(s)	10 ^(t)	10 ^(u)
5, 50	1.7530E+19 ^(v)	1.7056E+21 ^(w)	50 ^(x)	50 ^(y)
5, 100	2.6011E+25 ^(z)	3.0206E+27 ^(aa)	100 ^(ab)	100 ^(ac)
5, 150	1.8126E+29 ^(ad)	2.2501E+31 ^(ae)	150 ^(af)	150 ^(ag)
5, 200	1.1439E+32 ^(ah)	1.4707E+34 ^(ai)	200 ^(aj)	200 ^(ak)
5, 250	1.8314E+34 ^(al)	2.4062E+36 ^(am)	250 ^(an)	250 ^(ao)
5, 300	1.2054E+36 ^(ap)	1.6072E+38 ^(aq)	300 ^(ar)	300 ^(as)
5, 350	4.2539E+37 ^(at)	?	350 ^(au)	350 ^(av)
10, 10	4.2634E+13 ^(aw)	2.6754E+15 ^(ax)	10 ^(ay)	10 ^(az)
10, 50	1.3419E+40 ^(ba)	3.0801E+42 ^(bb)	50 ^(bc)	50 ^(bd)
10, 100	4.5274E+58 ^(be)	1.5562E+61 ^(bf)	100 ^(bg)	100 ^(bh)
10, 150	2.4252E+71 ^(bi)	?	150 ^(bj)	150 ^(bk)
10, 200	1.3861E+81 ^(bl)	?	200 ^(bm)	200 ^(bn)
20, 10	3.2308E+19 ^(bo)	2.3414E+21 ^(bp)	10 ^(bq)	10 ^(br)
20, 50	7.9113E+66 ^(bs)	2.6113E+69 ^(bt)	50 ^(bu)	50 ^(bv)
20, 100	1.6334E+107 ^(bw)	?	100 ^(bx)	100 ^(by)
20, 150	2.5424E+138 ^(bz)	?	150 ^(ca)	150 ^(cb)
30, 10	1.0100E+23 ^(cc)	7.6041E+24 ^(cd)	10 ^(ce)	10 ^(cf)
30, 50	6.4510E+83 ^(cg)	?	50 ^(ch)	50 ^(ci)
30, 100	5.7466E+139 ^(cj)	?	100 ^(ck)	100 ^(cl)
30, 150	?	?	?	150
40, 10	3.1162E+25 ^(cm)	?	10 ^(cn)	10 ^(co)
40, 50	1.1273E+96 ^(cp)	?	50 ^(cq)	50 ^(cr)
40, 100	?	?	?	100
40, 150	?	?	?	150
50, 10	2.6757E+27 ^(cs)	?	10 ^(ct)	10 ^(cu)
50, 50	?	?	?	50
50, 100	?	?	?	100
50, 150	?	?	?	150

^(m) stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).

⁽ⁿ⁾ the definition of Nested-Unit Petri Nets (NUPN) is available from <http://mcc.lip6.fr/nupn.php>

^(o) stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).

^(p) stated by CÆSAR.BDD version 2.0 to be true on 21 instance(s) out of 29, and unknown on the remaining 8 instance(s).

^(q) number of initial tokens, because the net is conservative.

^(r) computed at MCC'2014 by Marcie, PNMCM, PNXDD, and Stratagem.

^(s) computed at MCC'2014 by Marcie.

Other properties

CPI (Covered by P-Invariants), CTI (Covered by T-Invariants). All places of this Petri net are N-bounded, i.e. all tokens can gather on any one place. The net enjoys some symmetries.

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- (t) computed at MCC'2014 by Marcie and PNMC.
 - (u) computed at MCC'2014 by Marcie and PNMC.
 - (v) computed at MCC'2014 by Marcie and PNMC.
 - (w) computed at MCC'2014 by Marcie.
 - (x) computed at MCC'2014 by Marcie and PNMC.
 - (y) computed at MCC'2014 by Marcie and PNMC.
 - (z) computed at MCC'2014 by Marcie and PNMC.
 - (aa) computed at MCC'2014 by Marcie.
 - (ab) computed at MCC'2014 by Marcie and PNMC.
 - (ac) computed at MCC'2014 by Marcie and PNMC.
 - (ad) computed at MCC'2014 by Marcie and PNMC.
 - (ae) computed at MCC'2014 by Marcie.
 - (af) computed at MCC'2014 by Marcie and PNMC.
 - (ag) computed at MCC'2014 by Marcie and PNMC.
 - (ah) computed at MCC'2014 by Marcie and PNMC.
 - (ai) computed at MCC'2014 by Marcie.
 - (aj) computed at MCC'2014 by Marcie and PNMC.
 - (ak) computed at MCC'2014 by Marcie and PNMC.
 - (al) computed at MCC'2014 by Marcie and PNMC.
 - (am) computed at MCC'2014 by Marcie.
 - (an) computed at MCC'2014 by Marcie and PNMC.
 - (ao) computed at MCC'2014 by Marcie and PNMC.
 - (ap) computed at MCC'2014 by Marcie and PNMC.
 - (aq) computed at MCC'2014 by Marcie.
 - (ar) computed at MCC'2014 by Marcie and PNMC.
 - (as) computed at MCC'2014 by Marcie and PNMC.
 - (at) computed at MCC'2014 by PNMC.
 - (au) computed at MCC'2014 by PNMC.
 - (av) computed at MCC'2014 by PNMC.
 - (aw) computed at MCC'2014 by Marcie and PNMC.
 - (ax) computed at MCC'2014 by Marcie.
 - (ay) computed at MCC'2014 by Marcie and PNMC.
 - (az) computed at MCC'2014 by Marcie and PNMC.
 - (ba) computed at MCC'2014 by Marcie and PNMC.
 - (bb) computed at MCC'2014 by Marcie.
 - (bc) computed at MCC'2014 by Marcie and PNMC.
 - (bd) computed at MCC'2014 by Marcie and PNMC.
 - (be) computed at MCC'2014 by Marcie and PNMC.
 - (bf) computed at MCC'2014 by Marcie.
 - (bg) computed at MCC'2014 by Marcie and PNMC.
 - (bh) computed at MCC'2014 by Marcie and PNMC.
 - (bi) computed at MCC'2014 by Marcie and PNMC.
 - (bj) computed at MCC'2014 by Marcie and PNMC.
 - (bk) computed at MCC'2014 by Marcie and PNMC.
 - (bl) computed at MCC'2014 by PNMC.
 - (bm) computed at MCC'2014 by PNMC.
 - (bn) computed at MCC'2014 by PNMC.
 - (bo) computed at MCC'2014 by Marcie, PNMC, and PNXDD.
 - (bp) computed at MCC'2014 by Marcie.
 - (bq) computed at MCC'2014 by Marcie and PNMC.
 - (br) computed at MCC'2014 by Marcie and PNMC.
 - (bs) computed at MCC'2014 by Marcie and PNMC.
 - (bt) computed at MCC'2014 by Marcie.
 - (bu) computed at MCC'2014 by Marcie and PNMC.
 - (bv) computed at MCC'2014 by Marcie and PNMC.
 - (bw) computed at MCC'2014 by PNMC.
 - (bx) computed at MCC'2014 by PNMC.

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- (by) computed at MCC'2014 by PNMIC.
 - (bz) computed at MCC'2014 by PNMIC.
 - (ca) computed at MCC'2014 by PNMIC.
 - (cb) computed at MCC'2014 by PNMIC.
 - (cc) computed at MCC'2014 by Marcie and PNMIC.
 - (cd) computed at MCC'2014 by Marcie.
 - (ce) computed at MCC'2014 by Marcie and PNMIC.
 - (cf) computed at MCC'2014 by Marcie and PNMIC.
 - (cg) computed at MCC'2014 by PNMIC.
 - (ch) computed at MCC'2014 by PNMIC.
 - (ci) computed at MCC'2014 by PNMIC.
 - (ej) computed at MCC'2014 by PNMIC.
 - (ck) computed at MCC'2014 by PNMIC.
 - (cl) computed at MCC'2014 by PNMIC.
 - (cm) computed at MCC'2014 by PNMIC.
 - (cn) computed at MCC'2014 by PNMIC.
 - (co) computed at MCC'2014 by PNMIC.
 - (cp) computed at MCC'2014 by PNMIC.
 - (cq) computed at MCC'2014 by PNMIC.
 - (cr) computed at MCC'2014 by PNMIC.
 - (cs) computed at MCC'2014 by PNMIC.
 - (ct) computed at MCC'2014 by PNMIC.
 - (cu) computed at MCC'2014 by PNMIC.