

This form is a summary description of the model entitled “SupplyChain” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.

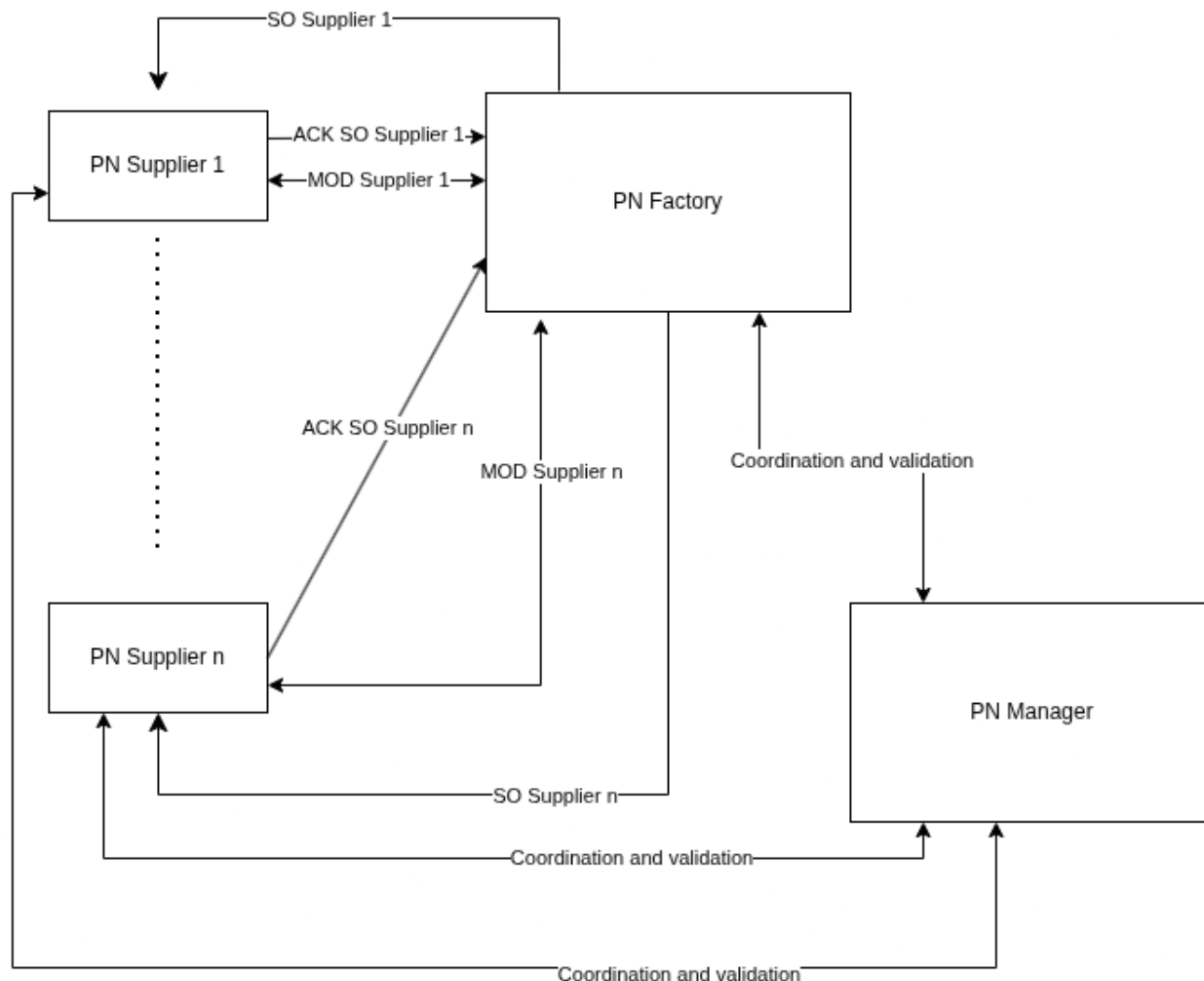
Description

This benchmark models a distributed supply chain as a composition of interacting Petri net components. The system consists of multiple suppliers, a central factory, and a shared manager coordinating validation and modification processes. Each component is specified as an individual Petri net, and the global model is obtained through a synchronous product based on shared transition labels.

The model captures resource-sharing constraints and coordination mechanisms between concurrent actors. In particular, the manager is represented as a critical shared resource whose limited availability induces contention when multiple suppliers compete for validation or modification, potentially leading to throughput limitations and structural bottlenecks.

The benchmark is parametric in the number of suppliers, allowing the exploration of different configurations and their impact on the system’s behaviour. It is designed to study the effects of resource contention, synchronisation, and concurrency on the global dynamics of the system.

The model originates from a timed and modular supply-chain model based on Time Petri Nets [1], from which timing constraints have been abstracted to obtain a purely untimed Petri net suitable for structural and behavioural analysis. The *timeout* transition was also removed.



Graphical representation of our overall system

References

1. Éric Lubat, Pierre-Emmanuel Hladik, Yoann Mateu, Rémi Sauvère. Modelling and Analysis of Supply Chains using Product Time Petri Nets. 7th Workshop on Models for Formal Analysis of Real Systems, Apr 2026, Turin (Italie), Italy. pp.23-39, ⟨10.4204/EPTCS.443.3⟩. ⟨hal-05585648⟩

Scaling parameter

Parameter name	Parameter description	Chosen parameter values
Number of suppliers N .	The benchmark is parametric in the number of suppliers and managers, allowing the exploration of different configurations and their impact on the system's behaviour. In this case, our only parameter is the number of suppliers.	2, 5, 10, 20, 50, 100, 200, 500, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, 10000

Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
$N = 2$	30	19	78
$N = 5$	69	43	186
$N = 10$	134	83	366
$N = 20$	264	163	726
$N = 50$	654	403	1806
$N = 100$	1304	803	3606
$N = 200$	2604	1603	7206
$N = 500$	6504	4003	18006
$N = 1000$	13004	8003	36006
$N = 2000$	26004	16003	72006
$N = 3000$	39004	24003	108006
$N = 4000$	52004	32003	144006
$N = 5000$	65004	40003	180006
$N = 6000$	78004	48003	216006
$N = 7000$	91004	56003	252006
$N = 8000$	104004	64003	288006
$N = 9000$	117004	72003	324006
$N = 10000$	130004	80003	360006

Structural properties

- ordinary** — all arcs have multiplicity one ✗ (a)
simple free choice — all transitions sharing a common input place have no other input place ✗ (b)
extended free choice — all transitions sharing a common input place have the same input places ✗ (c)
state machine — every transition has exactly one input place and exactly one output place ✗ (d)
marked graph — every place has exactly one input transition and exactly one output transition ✗ (e)
connected — there is an undirected path between every two nodes (places or transitions) ✓ (f)

(a) transition t_0 is weighted in end.net.

(b) the net is not ordinary.

(c) the net is not ordinary.

(d) see the *SYNC* transitions in Factory.

(e) see the *MOD* transitions in Supplier.

(f) stated by [CÆSAR.BDD](#) version 3.7 on all 18 instances (18 values of N).

- strongly connected** — *there is a directed path between every two nodes (places or transitions)* ✗ (g)
- source place(s)** — *one or more places have no input transitions* ✓ (h)
- sink place(s)** — *one or more places have no output transitions* ✗ (i)
- source transition(s)** — *one or more transitions have no input places* ✗ (j)
- sink transitions(s)** — *one or more transitions have no output places* ✗ (k)
- loop-free** — *no transition has an input place that is also an output place* ✗ (l)
- conservative** — *for each transition, the number of input arcs equals the number of output arcs* ✗ (m)
- subconservative** — *for each transition, the number of input arcs equals or exceeds the number of output arcs* ✓ (n)
- nested units** — *places are structured into hierarchically nested sequential units*^(o) ✗

Behavioural properties

- safe** — *in every reachable marking, there is no more than one token on a place* ✗ (p)
- dead place(s)** — *one or more places have no token in any reachable marking* ✓ (q)
- dead transition(s)** — *one or more transitions cannot fire from any reachable marking* ✓ (r)
- deadlock** — *there exists a reachable marking from which no transition can be fired* ✗
- reversible** — *from every reachable marking, there is a transition path going back to the initial marking* ✗
- live** — *for every transition t , from every reachable marking, one can reach a marking in which t can fire* ?

Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
$N = 2$	50	109	2	7 ^(s)
$N = 5$	14258	73443	2	13 ^(t)
$N = 10$	1.6124e+8	1.5620e+9	2	23 ^(u)
$N = 20$	1.5843e+16	2.9046e+17	2	43 ^(v)
$N = 50$	7.5440e+39	3.2893e+41	2	103 ^(w)
$N = 100$	4.8833e+79	4.1795e+81	2	203 ^(x)
$N = 200$	1.5795e+164	2.7066e+166	2	403 ^(y)
$N = 500$	5.3448e+417	2.3046e+420	2	1003 ^(z)
$N = 1000$?	?	?	2003 ^(aa)
$N = 2000$?	?	?	4003 ^(ab)
$N = 3000$?	?	?	6003 ^(ac)
$N = 4000$?	?	?	8003 ^(ad)
$N = 5000$?	?	?	10003 ^(ae)
$N = 6000$?	?	?	12003 ^(af)
$N = 7000$?	?	?	14003 ^(ag)
$N = 8000$?	?	?	16003 ^(ah)
$N = 9000$?	?	?	18003 ^(ai)
$N = 10000$?	?	?	20003 ^(aj)

(g) stated by [CÆSAR.BDD](#) version 3.7 on all 18 instances (18 values of N).

(h) *IDLE* places are sources.

(i) *SYNC* transitions are merged in end.net.

(j) stated by [CÆSAR.BDD](#) version 3.7 on all 18 instances (18 values of N).

(k) stated by [CÆSAR.BDD](#) version 3.7 on all 18 instances (18 values of N).

(l) *success* transitions in end.net.

(m) only output of *sync* is in end.net.

(n) only output of *sync* is in end.net.

(o) the definition of Nested-Unit Petri Nets (NUPN) is available from <http://mcc.lip6.fr/nupn.php>

(p) in end.net we need 2 tokens.

(q) we do not have to ask for *MOD*.

(r) we do not have to ask for *MOD*.

(s) number of initial tokens, because the net is sub-conservative.

Other properties

A supply-chain is viable when the success property eventually holds forever, i.e., when the final transition of the system is a **success**. Our *self* formulas can be expressed both textually and in classical LTL notation as follows:

Success property: Does the success transition eventually hold forever? In LTL:

$$\langle \rangle [] success$$

-
- (t) number of initial tokens, because the net is sub-conservative.
 - (u) number of initial tokens, because the net is sub-conservative.
 - (v) number of initial tokens, because the net is sub-conservative.
 - (w) number of initial tokens, because the net is sub-conservative.
 - (x) number of initial tokens, because the net is sub-conservative.
 - (y) number of initial tokens, because the net is sub-conservative.
 - (z) number of initial tokens, because the net is sub-conservative.
 - (aa) number of initial tokens, because the net is sub-conservative.
 - (ab) number of initial tokens, because the net is sub-conservative.
 - (ac) number of initial tokens, because the net is sub-conservative.
 - (ad) number of initial tokens, because the net is sub-conservative.
 - (ae) number of initial tokens, because the net is sub-conservative.
 - (af) number of initial tokens, because the net is sub-conservative.
 - (ag) number of initial tokens, because the net is sub-conservative.
 - (ah) number of initial tokens, because the net is sub-conservative.
 - (ai) number of initial tokens, because the net is sub-conservative.
 - (aj) number of initial tokens, because the net is sub-conservative.