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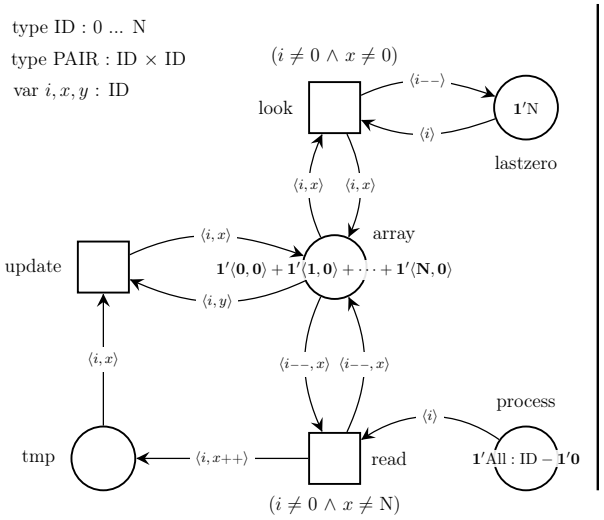
This form is a summary description of the model entitled "LastZero" proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.

Description

We give a symmetric net interpretation of the multi-threaded program "lastzero", that has been proposed as a benchmark for dynamic partial-order methods [1,2].

The program, displayed on the figure below (right), has N+1 processes operating on an array of N+1 elements, which are all initially zero (place *array*). In this program, process 0 iteratively searches the array for the highest index with value zero, starting from N and going backwards (transition *look*). The result is the final value in place *lastzero*. The other processes, say j (with $j \geq 1$), update the value at their assigned index on the array. It reads the value of the array at index $j-1$ and writes the value, incremented by one, at index j. Since the update operation is not atomic, it must be split in two: first, reading the value `array[j-1]` and storing it in a temporary variable (transition *read*), and then updating the value at `array[j]` with this tmp value (transition *update*).



Shared variables: `int array[0..N] := {0, 0, ..., 0};`

Process(0): `for (int i := N; array[i] != 0; i--);`

Process(j): `array[j] := array[j-1] + 1;`
($j \in 1..N$)

Graphical representation of LastZero-COL-N (left) and pseudocode for the program lastzero(N) (right).

References

1. Parosh Aziz Abdulla, Stavros Aronis, Bengt Jonsson, Konstantinos Sagonas (2017). *Source Sets: A Foundation for Optimal Dynamic Partial Order Reduction.* Journal of the ACM (JACM), Volume 64, Issue 4 Article No.: 25.

2. Frédéric Herbreteau, Sarah Larroze-Jardiné, Gérald Point, Igor Walukiewicz (2024). *Revisiting Stateful Partial-Order Reduction.* CoRR abs/2411.16921.

Scaling parameter

Parameter name	Parameter description	Chosen parameter values
N	N is the number of processes that update the array. The size of the array is N+1.	8,10,12,14,16,18,20,22,24,26

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Size of the colored net model

number of places:	4
number of transitions:	3
number of arcs:	11

Size of the derived P/T model instances

Parameter	Number of places	Number of transitions	Number of arcs
LastZero-PT-NX	$2\,X^2 + 6\,X + 4$	$X^3 + 5.X^2 + 3.X + 1$	$3\,X^3 + 17\,X^2 + 9\,X + 3$
$N = 8$	180	857	2 699
$N = 10$	264	1 531	4 793
$N = 12$	364	2 485	7 743
$N = 14$	480	3 767	11 693
$N = 16$	612	5 425	16 787
$N = 18$	760	7 507	23 169
$N = 20$	924	10 061	30 983
$N = 22$	1 104	13 135	40 373
$N = 24$	1 300	16 777	51 483
$N = 26$	1 512	21 035	64 457

Structural properties

ordinary — *all arcs have multiplicity one* ..... ✔
simple free choice — *all transitions sharing a common input place have no other input place* ..... ✘ [a]
extended free choice — *all transitions sharing a common input place have the same input places* ..... ✘ [b]
state machine — *every transition has exactly one input place and exactly one output place* ..... ✘ [c]
marked graph — *every place has exactly one input transition and exactly one output transition* ..... ✘ [d]
connected — *there is an undirected path between every two nodes (places or transitions)* ..... ✘ [e]
strongly connected — *there is a directed path between every two nodes (places or transitions)* ..... ✘ [f]
source place(s) — *one or more places have no input transitions* ..... ✔ [g]
sink place(s) — *one or more places have no output transitions* ..... ✔ [h]
source transition(s) — *one or more transitions have no input places* ..... ✘ [i]
sink transitions(s) — *one or more transitions have no output places* ..... ✘ [j]
loop-free — *no transition has an input place that is also an output place* ..... ✘ [k]
conservative — *for each transition, the number of input arcs equals the number of output arcs* ..... ✘ [l]
subconservative — *for each transition, the number of input arcs equals or exceeds the number of output arcs* ..... ✔ [m]
nested units — *places are structured into hierarchically nested sequential units* [n] ..... ✘

[a] stated by CÆSAR.BDD version 3.5 on all 4 instances (8, 10, 12, 14, 16, 18, 20, 22, 24 and 26).
[b] stated by CÆSAR.BDD version 3.5 on all 4 instances (8, 10, 12, 14, 16, 18, 20, 22, 24 and 26).
[c] stated by CÆSAR.BDD version 3.5 on all 4 instances (8, 10, 12, 14, 16, 18, 20, 22, 24 and 26).
[d] stated by CÆSAR.BDD version 3.5 on all 4 instances (8, 10, 12, 14, 16, 18, 20, 22, 24 and 26).
[e] stated by CÆSAR.BDD version 3.5 on all 4 instances (8, 10, 12, 14, 16, 18, 20, 22, 24 and 26).
[f] the net is not connected and, thus, not strongly connected.
[g] place *process* has no input transition. It is also the case for all the PT places of the form $tmp\_\langle 0, i\rangle$.
[h] even though the colored net has no "structural" sink place, the PT place ''*lastzero*_0' has no output transitions.
[i] stated by CÆSAR.BDD version 3.5 on all 4 instances (8, 10, 12, 14, 16, 18, 20, 22, 24 and 26).
[j] stated by CÆSAR.BDD version 3.5 on all 4 instances (8, 10, 12, 14, 16, 18, 20, 22, 24 and 26).
[k] stated by CÆSAR.BDD version 3.5 on all 4 instances (8, 10, 12, 14, 16, 18, 20, 22, 24 and 26).
[l] stated by CÆSAR.BDD version 3.5 on all 4 instances (8, 10, 12, 14, 16, 18, 20, 22, 24 and 26).
[m] stated by CÆSAR.BDD version 3.5 on all 4 instances (8, 10, 12, 14, 16, 18, 20, 22, 24 and 26).
[n] the definition of Nested-Unit Petri Nets (NUPN) is available from http://mcc.lip6.fr/nupn.php

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Behavioural properties

safe — *in every reachable marking, there is no more than one token on a place* ..... ✔
dead place(s) — *one or more places have no token in any reachable marking* ..... ✔ [o]
dead transition(s) — *one or more transitions cannot fire from any reachable marking* ..... ✔ [p]
deadlock — *there exists a reachable marking from which no transition can be fired* ..... ✔
reversible — *from every reachable marking, there is a transition path going back to the initial marking* ..... ✘
live — *for every transition t, from every reachable marking, one can reach a marking in which t can fire* ..... ✘ [q]

Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
$N = 8$	52 919 [r]	215 980 [s]	1 [t]	18 [u]
$N = 10$	738 761	3 747 657	1	22
$N = 12$	10 296 391	62 497 250	1	26
$N = 14$	143 437 337	1 013 922 659	1	30
$N = 16$	1 997 932 823	1.61200E+10	1	34
$N = 18$	2.78280E+10	2.52352E+11	1	38
$N = 20$	?	?	1	42
$N = 22$	?	?	1	46
$N = 24$	?	?	1	50
$N = 26$	?	?	1	54

Other properties

A first property is termination, since every process must eventually update its associated value in the array, and therefore computation must always eventually terminate. The associated CTL formula is:

$$\text{END} \ : \ \text{AG AF deadlock}$$

A very unlikely situation is when the lastzero index is 0, which occurs only if all the processes i, with $i \neq 0$, update their values after process 0 ends its loop. Another very unlikely scenario is when `array[N] = N`. Indeed, the value in `array[i]` is always bounded by i since, in the worst case, all the processes with identity $j \in 0..i$ have updated their values before process i (and in order). Therefore, it is possible, but highly unlikely, that both *lastzero* equals 0 and `array[N] = N`. We can describe this situation using a simple reachability property, only with PT models. For instance, in the case $N = 3$ [v]:

$$\text{UNLIKELY} \ : \ \text{EF}\left((\text{lastzero\_1} = 1) \wedge (\text{array\_4\_4} = 1)\right)$$

[o] place $tmp\_\langle 0, i\rangle$ is dead for all $i \in 0..N$, which corresponds to the fact that no process updates `array[0]`.
[p] stated by CÆSAR.BDD version 3.5 on all 4 instances (8, 10, 12, 14, 16, 18, 20, 22, 24 and 26).
[q] the net has dead transitions.
[r] computed by TINA version 3.9.0 on February 2026.
[s] computed by TINA version 3.9.0 on February 2026.
[t] the net is safe; confirmed by TINA version 3.9.0 on February 2026.
[u] since LastZero is subconservative, the maximum total number of tokens is observed in the initial marking, and is equal to $2\,N + 2$; confirmed by TINA version 3.9.0 on February 2026.
[v] We encode values of type ID with integers in the range 1..N+1.