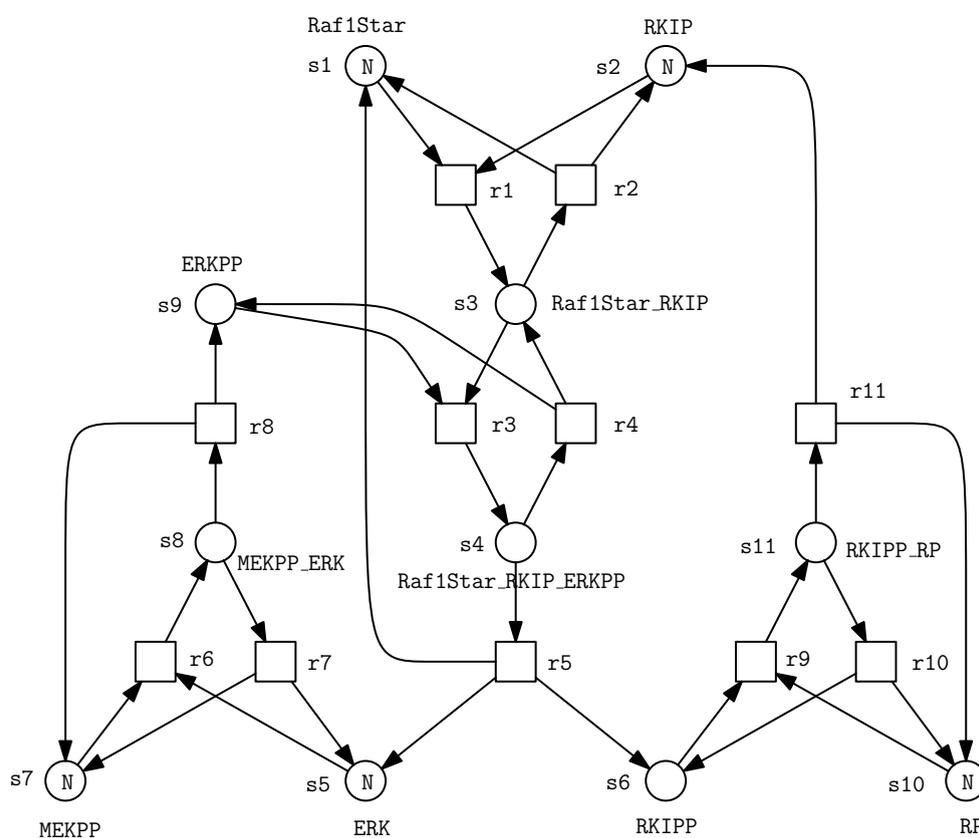


*This form is a summary description of the model entitled “ERK” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.*

## Description

ERK is a short name for “RKIP/MEK-ERK signalling pathway”. The RKIP inhibited ERK pathway published in [CSK+03], discussed as qualitative and continuous Petri nets in [GH06], and as three related Petri net models in [HDG10].

*In March 2020, Pierre Bowvier and Hubert Garavel provided a decomposition of the only one-safe instance of this model into a network of communicating automata. This network is expressed as a Nested-Unit Petri Net (NUPN) that can be found in the “toolspecific” section of the corresponding PNML file.*



Graphical representation with parameter  $N$

## References

- CSK+03** K.-H. Cho, S.-Y. Shin, H.-W. Kim, O. Wolkenhauer, B. McFerran and W. Kolch: Mathematical modeling of the influence of RKIP on the ERK signaling pathway; In CMSB 2003, LNCS 2602, Springer, pages 127–141, 2003.
- GH06** Gilbert, D. and Heiner, M.: From Petri nets to differential equations - an integrative approach for biochemical network analysis; In Proc. ICATPN 2006, LNCS 4024, Springer, pages 181–200, 2006.
- HDG10** M. Heiner, R. Donaldson and D. Gilbert: Petri Nets for Systems Biology; In Symbolic Systems Biology: Theory and Methods, (MS Iyengar, Ed.), Jones & Bartlett Learning, LCC, pages 61–97, 2010.

## Scaling parameter

Parameter name	Parameter description	Chosen parameter values
$N$	initial number of tokens on places ERK, MEKPP, Raf1Star, RKIP and RP	1, 10, 100, 1000, 10000, 100000

## Size of the model

Parameter	Number of places	Number of transitions	Number of arcs	Number of units	HWB code
$N = 1$	11	11	34	6	1-5-8
$N = 10$	11	11	34	-	--11
$N = 100$	11	11	34	-	--11
$N = 1000$	11	11	34	-	--11
$N = 10000$	11	11	34	-	--11
$N = 100000$	11	11	34	-	--11

## Structural properties

<b>ordinary</b> — all arcs have multiplicity one .....	yes
<b>simple free choice</b> — all transitions sharing a common input place have no other input place .....	no <sup>(a)</sup>
<b>extended free choice</b> — all transitions sharing a common input place have the same input places .....	no <sup>(b)</sup>
<b>state machine</b> — every transition has exactly one input place and exactly one output place .....	no <sup>(c)</sup>
<b>marked graph</b> — every place has exactly one input transition and exactly one output transition .....	no <sup>(d)</sup>
<b>connected</b> — there is an undirected path between every two nodes (places or transitions) .....	yes <sup>(e)</sup>
<b>strongly connected</b> — there is a directed path between every two nodes (places or transitions) .....	yes <sup>(f)</sup>
<b>source place(s)</b> — one or more places have no input transitions .....	no <sup>(g)</sup>
<b>sink place(s)</b> — one or more places have no output transitions .....	no <sup>(h)</sup>
<b>source transition(s)</b> — one or more transitions have no input places .....	no <sup>(i)</sup>
<b>sink transition(s)</b> — one or more transitions have no output places .....	no <sup>(j)</sup>
<b>loop-free</b> — no transition has an input place that is also an output place .....	yes <sup>(k)</sup>
<b>conservative</b> — for each transition, the number of input arcs equals the number of output arcs .....	no <sup>(l)</sup>
<b>subconservative</b> — for each transition, the number of input arcs equals or exceeds the number of output arcs .....	no <sup>(m)</sup>
<b>nested units</b> — places are structured into hierarchically nested sequential units <sup>(n)</sup> .....	? <sup>(o)</sup>

## Behavioural properties

<b>safe</b> — in every reachable marking, there is no more than one token on a place .....	? <sup>(p)</sup>
<b>dead place(s)</b> — one or more places have no token in any reachable marking .....	no <sup>(q)</sup>

<sup>(a)</sup> there is an arc from place “Raf1Star\_RKIP” (which has 2 outgoing transitions) to transition “r3” (which has 2 input places).

<sup>(b)</sup> transitions “r3” and “r2” share a common input place “Raf1Star\_RKIP”, but only the former transition has input place “ERKPP”.

<sup>(c)</sup> 11 transitions are not of a state machine, e.g., transition “r1”.

<sup>(d)</sup> stated by CÆSAR.BDD version 3.3 on all 6 instances (1, 10, 10<sup>2</sup>, 10<sup>3</sup>, 10<sup>4</sup>, and 10<sup>5</sup>).

<sup>(e)</sup> stated by CÆSAR.BDD version 2.0 on all 6 instances (1, 10, 10<sup>2</sup>, 10<sup>3</sup>, 10<sup>4</sup>, and 10<sup>5</sup>).

<sup>(f)</sup> stated by CÆSAR.BDD version 2.0 on all 6 instances (1, 10, 10<sup>2</sup>, 10<sup>3</sup>, 10<sup>4</sup>, and 10<sup>5</sup>).

<sup>(g)</sup> stated by CÆSAR.BDD version 2.0 on all 6 instances (1, 10, 10<sup>2</sup>, 10<sup>3</sup>, 10<sup>4</sup>, and 10<sup>5</sup>).

<sup>(h)</sup> stated by CÆSAR.BDD version 2.0 on all 6 instances (1, 10, 10<sup>2</sup>, 10<sup>3</sup>, 10<sup>4</sup>, and 10<sup>5</sup>).

<sup>(i)</sup> stated by CÆSAR.BDD version 2.0 on all 6 instances (1, 10, 10<sup>2</sup>, 10<sup>3</sup>, 10<sup>4</sup>, and 10<sup>5</sup>).

<sup>(j)</sup> stated by CÆSAR.BDD version 2.0 on all 6 instances (1, 10, 10<sup>2</sup>, 10<sup>3</sup>, 10<sup>4</sup>, and 10<sup>5</sup>).

<sup>(k)</sup> stated by CÆSAR.BDD version 2.0 on all 6 instances (1, 10, 10<sup>2</sup>, 10<sup>3</sup>, 10<sup>4</sup>, and 10<sup>5</sup>).

<sup>(l)</sup> 11 transitions are not conservative, e.g., transition “r1”.

<sup>(m)</sup> 7 transitions are not subconservative, e.g., transition “r2”.

<sup>(n)</sup> the definition of Nested-Unit Petri Nets (NUPN) is available from <http://mcc.lip6.fr/nupn.php>

<sup>(o)</sup> stated by CÆSAR.BDD version 3.3 to be true on 1 instance(s) out of 6, and false on the remaining 5 instance(s).

<sup>(p)</sup> stated by CÆSAR.BDD version 2.0 to be true for  $N = 1$ , and false on the remaining 5 instance(s).

<sup>(q)</sup> stated by CÆSAR.BDD version 3.3 on all 6 instances (1, 10, 10<sup>2</sup>, 10<sup>3</sup>, 10<sup>4</sup>, and 10<sup>5</sup>).

**dead transition(s)** — *one or more transitions cannot fire from any reachable marking* ..... no <sup>(r)</sup>  
**deadlock** — *there exists a reachable marking from which no transition can be fired* .....no <sup>(s)</sup>  
**reversible** — *from every reachable marking, there is a transition path going back to the initial marking* ..... yes  
**live** — *for every transition  $t$ , from every reachable marking, one can reach a marking in which  $t$  can fire* ..... yes

## Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
$N = 1$	13 <sup>(t)</sup>	30 <sup>(u)</sup>	$N$ <sup>(v)</sup>	$5 \cdot N$ <sup>(w)</sup>
$N = 10$	47 047 <sup>(x)</sup>	372 372 <sup>(y)</sup>	$N$ <sup>(z)</sup>	$5 \cdot N$ <sup>(aa)</sup>
$N = 100$	1.5914E+10 <sup>(ab)</sup>	1.68445E+11 <sup>(ac)</sup>	$N$ <sup>(ad)</sup>	$5 \cdot N$ <sup>(ae)</sup>
$N = 1\,000$	14 081 614 073 878 351 <sup>(af)</sup>	?	$N$	$5 \cdot N$
$N = 10\,000$	?	?	$N$	$5 \cdot N$
$N = 100\,000$	?	?	$N$	$5 \cdot N$

<sup>(r)</sup> stated by [CÆSAR.BDD](#) version 2.0 on all 6 instances (1, 10, 10<sup>2</sup>, 10<sup>3</sup>, 10<sup>4</sup>, and 10<sup>5</sup>).

<sup>(s)</sup> confirmed by [CÆSAR.BDD](#) version 2.0 to be false for  $N = 1$ , and unknown on the remaining 5 instance(s); confirmed at MCC'2014 by Lola on all 6 instances and Tapaal on 2 instances.

<sup>(t)</sup> confirmed at MCC'2014 by Marcie, PNMC, PNXDD, Stratagem, and Tapaal.

<sup>(u)</sup> computed at MCC'2014 by Marcie.

<sup>(v)</sup> confirmed at MCC'2014 by Marcie, PNMC, and Tapaal.

<sup>(w)</sup> confirmed at MCC'2014 Marcie, PNMC, and Tapaal.

<sup>(x)</sup> computed by Marcie on 2013-12-13; confirmed at MCC'2014 by Marcie, PNMC, PNXDD, Stratagem, and Tapaal.

<sup>(y)</sup> computed at MCC'2014 by Marcie.

<sup>(z)</sup> confirmed at MCC'2014 by Marcie, PNMC, and Tapaal.

<sup>(aa)</sup> confirmed at MCC'2014 by Marcie, PNMC, and Tapaal.

<sup>(ab)</sup> exact value 15 914 114 086 computed by Marcie on 2013-12-13; confirmed at MCC'2014 by Marcie and PNMC.

<sup>(ac)</sup> computed at MCC'2014 by Marcie.

<sup>(ad)</sup> confirmed at MCC'2014 by Marcie and PNMC.

<sup>(ae)</sup> confirmed at MCC'2014 by Marcie and PNMC.

<sup>(af)</sup> computed by Marcie on 2013-12-13.