

*This form is a summary description of the model entitled “DiscoveryGPU” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.*

## Description

This model describes the discovery protocol [1] used to dynamically detect the amount of thread workgroups that can be scheduled at the same time on a given GPU, for a given application. Each workgroup participate in a mutex-protected poll, which is closed once a workgroup is able to re-visit the poll. All workgroups that were able to participate in the poll are thus actually running in parallel on different hardware-level compute-units. As GPUs have non-preemptive scheduling, these discovered workgroups are able to safely participate in blocking synchronisation. This protocol is critical to enable safe inter-workgroup blocking synchronisation, which in turn can provide significant performance improvements in some applications.

The discovery protocol was formally specified using the [LNT](#) value-passing process calculus and analyzed using the verification tools available in the [CADP](#) toolbox. The collection of P/T nets was obtained from the LNT specifications of the protocol. Each LNT specification was translated automatically to [LOTOS](#), and then to an interpreted Petri net using the [CADP](#) toolbox. Finally, a P/T net was obtained by stripping out all data-related information (variables, types, assignments, guards, etc.) from the interpreted Petri net, leading to a NUPN (Nested-Unit Petri Net) model translated to PNML using the [CÆSAR.BDD](#) tool.

Each instance of the model is parameterized by the number  $N$  of threads.

Each instance is also parameterized by its version  $V$ , which specifies how the NUPN has been produced from the LOTOS specification.  $V$  is either equal to “ $a$ ” if the NUPN has been generated *after* applying all the structural and data-flow optimizations of the [CÆSAR](#) compiler for LOTOS, or to “ $b$ ” if the NUPN has been generated *before* these optimizations.

## References

[1] Tyler Sorensen, Alastair F. Donaldson, Mark Batty, Ganesh Gopalakrishnan, and Zvonimir Rakamaric. Portable Inter-workgroup Barrier Synchronisation for GPUs. In Proc. of the 31st Annual ACM SIGPLAN Conference on Object-Oriented Programming, Systems, Languages, and Applications (OOPSLA’16), Amsterdam, The Netherlands, November 2016.

## Scaling parameter

Parameter name	Parameter description	Chosen parameter values
$(N, V)$	$N$ is the number of threads and $V$ is the version defined above	$\{6, \dots, 15\} \times \{a, b\}$

## Size of the model

Parameter	Number of places	Number of transitions	Number of arcs	Number of units	HWB code
$N = 06, V = a$	63	85	273	9	2-8-27
$N = 06, V = b$	184	194	503	15	7-8-44
$N = 07, V = a$	73	99	318	10	2-9-31
$N = 07, V = b$	212	224	582	17	8-9-50
$N = 08, V = a$	83	113	363	11	2-10-35
$N = 08, V = b$	240	254	661	19	9-10-56
$N = 09, V = a$	93	127	408	12	2-11-39
$N = 09, V = b$	268	284	740	21	10-11-62
$N = 10, V = a$	103	141	453	13	2-12-43
$N = 10, V = b$	296	314	819	23	11-12-68
$N = 11, V = a$	113	155	498	14	2-13-47
$N = 11, V = b$	324	344	898	25	12-13-74
$N = 12, V = a$	123	169	543	15	2-14-51
$N = 12, V = b$	352	374	977	27	13-14-80
$N = 13, V = a$	133	183	588	16	2-15-55
$N = 13, V = b$	380	404	1056	29	14-15-86
$N = 14, V = a$	143	197	633	17	2-16-59
$N = 14, V = b$	408	434	1135	31	15-16-92
$N = 15, V = a$	153	211	678	18	2-17-63
$N = 15, V = b$	436	464	1214	33	16-17-98

## Structural properties

<b>ordinary</b> — all arcs have multiplicity one .....	yes
<b>simple free choice</b> — all transitions sharing a common input place have no other input place .....	no <sup>(a)</sup>
<b>extended free choice</b> — all transitions sharing a common input place have the same input places .....	no <sup>(b)</sup>
<b>state machine</b> — every transition has exactly one input place and exactly one output place .....	no <sup>(c)</sup>
<b>marked graph</b> — every place has exactly one input transition and exactly one output transition .....	no <sup>(d)</sup>
<b>connected</b> — there is an undirected path between every two nodes (places or transitions) .....	yes <sup>(e)</sup>
<b>strongly connected</b> — there is a directed path between every two nodes (places or transitions) .....	no <sup>(f)</sup>
<b>source place(s)</b> — one or more places have no input transitions .....	yes <sup>(g)</sup>
<b>sink place(s)</b> — one or more places have no output transitions .....	? <sup>(h)</sup>
<b>source transition(s)</b> — one or more transitions have no input places .....	no <sup>(i)</sup>
<b>sink transitions(s)</b> — one or more transitions have no output places .....	? <sup>(j)</sup>
<b>loop-free</b> — no transition has an input place that is also an output place .....	? <sup>(k)</sup>
<b>conservative</b> — for each transition, the number of input arcs equals the number of output arcs .....	no <sup>(l)</sup>
<b>subconservative</b> — for each transition, the number of input arcs equals or exceeds the number of output arcs .....	no <sup>(m)</sup>
<b>nested units</b> — places are structured into hierarchically nested sequential units <sup>(n)</sup> .....	yes

<sup>(a)</sup> stated by [CÆSAR.BDD](#) version 2.7 on all 20 instances (10 values of  $N \times 2$  values of  $V$ ).

<sup>(b)</sup> stated by [CÆSAR.BDD](#) version 2.7 on all 20 instances (10 values of  $N \times 2$  values of  $V$ ).

<sup>(c)</sup> stated by [CÆSAR.BDD](#) version 2.7 on all 20 instances (10 values of  $N \times 2$  values of  $V$ ).

<sup>(d)</sup> stated by [CÆSAR.BDD](#) version 2.7 on all 20 instances (10 values of  $N \times 2$  values of  $V$ ).

<sup>(e)</sup> stated by [CÆSAR.BDD](#) version 2.7 on all 20 instances (10 values of  $N \times 2$  values of  $V$ ).

<sup>(f)</sup> from place 1 one cannot reach place 0.

<sup>(g)</sup> place 0 is a source place.

<sup>(h)</sup> stated by [CÆSAR.BDD](#) version 2.7 to be true on 10 instance(s) out of 20, and false on the remaining 10 instance(s).

<sup>(i)</sup> stated by [CÆSAR.BDD](#) version 2.7 on all 20 instances (10 values of  $N \times 2$  values of  $V$ ).

<sup>(j)</sup> stated by [CÆSAR.BDD](#) version 2.7 to be true on 10 instance(s) out of 20, and false on the remaining 10 instance(s).

<sup>(k)</sup> stated by [CÆSAR.BDD](#) version 2.7 to be true on 10 instance(s) out of 20, and false on the remaining 10 instance(s).

<sup>(l)</sup> stated by [CÆSAR.BDD](#) version 2.7 on all 20 instances (10 values of  $N \times 2$  values of  $V$ ).

<sup>(m)</sup> stated by [CÆSAR.BDD](#) version 2.7 on all 20 instances (10 values of  $N \times 2$  values of  $V$ ).

<sup>(n)</sup> the definition of Nested-Unit Petri Nets (NUPN) is available from <http://mcc.lip6.fr/nupn.php>

## Behavioural properties

- safe** — *in every reachable marking, there is no more than one token on a place* .....yes <sup>(o)</sup>  
**dead place(s)** — *one or more places have no token in any reachable marking* .....no <sup>(p)</sup>  
**dead transition(s)** — *one or more transitions cannot fire from any reachable marking* .....no <sup>(q)</sup>  
**deadlock** — *there exists a reachable marking from which no transition can be fired* .....? <sup>(r)</sup>  
**reversible** — *from every reachable marking, there is a transition path going back to the initial marking* .....? <sup>(s)</sup>  
**live** — *for every transition  $t$ , from every reachable marking, one can reach a marking in which  $t$  can fire* .....? <sup>(t)</sup>

## Size of the marking graphs

Parameter	Number of reach- able markings	Number of tran- sition firings	Max. number of tokens per place	Max. number of tokens per marking
$N = 06, V = a$	1.77156e+06 <sup>(u)</sup>	?	1	8
$N = 06, V = b$	6.13871e+09 <sup>(v)</sup>	?	1	8
$N = 07, V = a$	1.94872e+07 <sup>(w)</sup>	?	1	9
$N = 07, V = b$	$\geq 1.59038e+11$ <sup>(x)</sup>	?	1 <sup>(y)</sup>	9
$N = 08, V = a$	2.14359e+08 <sup>(z)</sup>	?	1	10
$N = 08, V = b$	$\geq 4.16622e+11$ <sup>(aa)</sup>	?	1 <sup>(ab)</sup>	10
$N = 09, V = a$	2.35795e+09 <sup>(ac)</sup>	?	1	11
$N = 09, V = b$	$\geq 1.44163e+12$ <sup>(ad)</sup>	?	1	11
$N = 10, V = a$	2.59374e+10 <sup>(ae)</sup>	?	1	12
$N = 10, V = b$	$\geq 5.47101e+12$ <sup>(af)</sup>	?	1 <sup>(ag)</sup>	12
$N = 11, V = a$	2.85312e+11 <sup>(ah)</sup>	?	1	13
$N = 11, V = b$	$\geq 1.61943e+13$ <sup>(ai)</sup>	?	1 <sup>(aj)</sup>	13
$N = 12, V = a$	3.13843e+12 <sup>(ak)</sup>	?	1	14
$N = 12, V = b$	$\geq 3.85496e+13$ <sup>(al)</sup>	?	1 <sup>(am)</sup>	14
$N = 13, V = a$	3.45227e+13 <sup>(an)</sup>	?	1	15
$N = 13, V = b$	$\geq 7.55166e+13$ <sup>(ao)</sup>	?	1 <sup>(ap)</sup>	15
$N = 14, V = a$	3.7975e+14 <sup>(aq)</sup>	?	1	16
$N = 14, V = b$	$\geq 1.23851e+14$ <sup>(ar)</sup>	?	1 <sup>(as)</sup>	16
$N = 15, V = a$	4.17725e+15 <sup>(at)</sup>	?	1	17
$N = 15, V = b$	$\geq 4.73641e+14$ <sup>(au)</sup>	?	1 <sup>(av)</sup>	17

- <sup>(o)</sup> safe by construction – stated by the [CÆSAR](#) compiler.  
<sup>(p)</sup> stated by [CÆSAR.BDD](#) version 3.3 on all 20 instances (10 values of  $N \times 2$  values of  $V$ ).  
<sup>(q)</sup> stated by [CÆSAR.BDD](#) version 2.7 on all 20 instances (10 values of  $N \times 2$  values of  $V$ ).  
<sup>(r)</sup> stated by [CÆSAR.BDD](#) version 2.7 to be true on 11 instance(s) out of 20, and unknown on the remaining 9 instance(s).  
<sup>(s)</sup> stated by [CÆSAR.BDD](#) version 2.7 to be false on 11 instance(s) out of 20, and unknown on the remaining 9 instance(s).  
<sup>(t)</sup> stated by [CÆSAR.BDD](#) version 2.7 to be false on 11 instance(s) out of 20, and unknown on the remaining 9 instance(s).  
<sup>(u)</sup> stated by [CÆSAR.BDD](#) version 2.7.  
<sup>(v)</sup> stated by [CÆSAR.BDD](#) version 2.7.  
<sup>(w)</sup> stated by [CÆSAR.BDD](#) version 2.7.  
<sup>(x)</sup> stated by [CÆSAR.BDD](#) version 2.7.  
<sup>(y)</sup> stated by the [CÆSAR](#) compiler.  
<sup>(z)</sup> stated by [CÆSAR.BDD](#) version 2.7.  
<sup>(aa)</sup> stated by [CÆSAR.BDD](#) version 2.7.  
<sup>(ab)</sup> stated by the [CÆSAR](#) compiler.  
<sup>(ac)</sup> stated by [CÆSAR.BDD](#) version 2.7.  
<sup>(ad)</sup> stated by [CÆSAR.BDD](#) version 2.7.  
<sup>(ae)</sup> stated by [CÆSAR.BDD](#) version 2.7.  
<sup>(af)</sup> stated by [CÆSAR.BDD](#) version 2.7.  
<sup>(ag)</sup> stated by the [CÆSAR](#) compiler.  
<sup>(ah)</sup> stated by [CÆSAR.BDD](#) version 2.7.  
<sup>(ai)</sup> stated by [CÆSAR.BDD](#) version 2.7.  
<sup>(aj)</sup> stated by the [CÆSAR](#) compiler.  
<sup>(ak)</sup> stated by [CÆSAR.BDD](#) version 2.7.  
<sup>(al)</sup> stated by [CÆSAR.BDD](#) version 2.7.  
<sup>(am)</sup> stated by the [CÆSAR](#) compiler.

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(an) stated by [CÆSAR.BDD](#) version 2.7.  
(ao) stated by [CÆSAR.BDD](#) version 2.7.  
(ap) stated by the [CÆSAR](#) compiler.  
(aq) stated by [CÆSAR.BDD](#) version 2.7.  
(ar) stated by [CÆSAR.BDD](#) version 2.7.  
(as) stated by the [CÆSAR](#) compiler.  
(at) stated by [CÆSAR.BDD](#) version 2.7.  
(au) stated by [CÆSAR.BDD](#) version 2.7.  
(av) stated by the [CÆSAR](#) compiler.