Elvio G. Amparore
amparore@di.unito.it

This form is a summary description of the model entitled "Sudoku" proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.

Description

The model represents the $N \times N$ Sudoku board, implementing the constraints of the Sudoku game. In the Sudoku puzzle, a $N \times N$ grid must be filled by symbols in $1 \dots N$. The grid is partitioned into N *regions*, that form an overlay of $R \times R$ sub-grids of size $R \times R$, with $N = R^2$. A valid Sudoku solution satisfies these constraints:

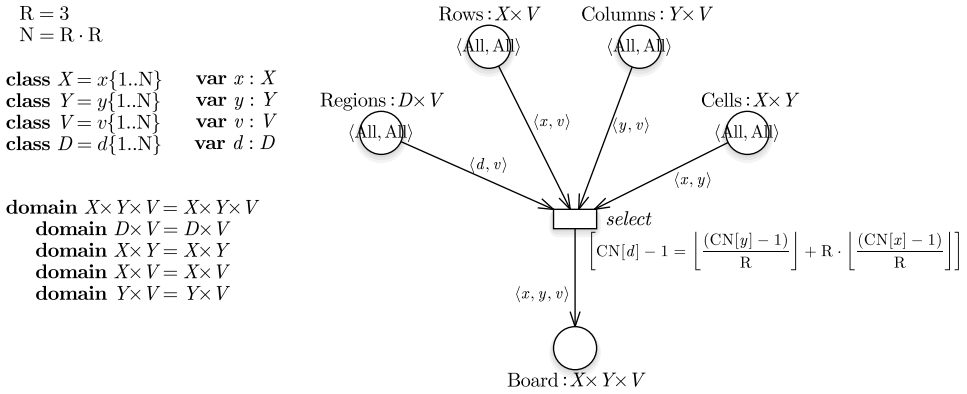
1. The same symbol can appear only once on each row;
2. The same symbol can appear only once on each column;
3. The same symbol can appear only once in each region;

There are two versions of this model:

- Version A is simplified and represents constraints 1 and 2;
- Version B represents all the constraints of the Sudoku game, and is defined only for values of N that are natural squares.

The board does not start with any initial assignment, therefore the statespace is the full exploration of all the possible boards that do not violate the game constraints. All valid Sudoku solutions correspond to the reachable markings where $token - count(\text{Cells} = 0)$.

In April 2021, Pierre Bouvier provided a decomposition of all instances of this model into networks of communicating automata. Each network is expressed as a Nested-Unit Petri Net (NUPN) that can be found, for each instance, in the "toolspecific" section of the corresponding PNML file.



Graphical representation for the $N \times N$ model, with $N = 9$.

References

Scaling parameter

Parameter name	Parameter description	Chosen parameter values
N	Side of the board	versions A and B, with N between 1 and 16

Elvio G. Amparore
amparore@di.unito.it

Size of the model

Parameter	Number of places	Number of transitions	Number of arcs	Number of units	HWB code
N (version A)	$3N^2 + N^3$	N^3	$4N^3$	$1 + 3N^2$	1–?–?
$N = 1$ (version A)	4	1	4	4	1–3–3
$N = 2$ (version A)	20	8	32	13	1–12–16
$N = 3$ (version A)	54	27	108	28	1–27–36
$N = 4$ (version A)	112	64	256	49	1–48–82
$N = 5$ (version A)	200	125	500	76	1–75–127
$N = 6$ (version A)	324	216	864	109	1–108–184
$N = 7$ (version A)	490	343	1372	148	1–147–247
$N = 8$ (version A)	704	512	2048	193	1–192–384
$N = 9$ (version A)	972	729	2916	244	1–243–494
$N = 10$ (version A)	1300	1000	4000	301	1–300–603
$N = 11$ (version A)	1694	1331	5324	364	1–363–726
$N = 12$ (version A)	2160	1728	6912	433	1–432–891
$N = 13$ (version A)	2704	2197	8788	508	1–507–1014
$N = 14$ (version A)	3332	2744	10976	589	1–588–1224
$N = 15$ (version A)	4050	3375	13500	676	1–675–1386
$N = 16$ (version A)	4864	4096	16384	769	1–768–1803
N (version B)	$4N^2 + N^3$	N^3	$5N^3$	$1 + 4N^2$	1–?–?
$N = 1$ (version B)	5	1	5	5	1–4–4
$N = 4$ (version B)	128	64	320	65	1–64–96
$N = 9$ (version B)	1053	729	3645	325	1–324–567
$N = 16$ (version B)	5120	4096	20480	1025	1–1024–2054

Structural properties

ordinary — *all arcs have multiplicity one* ..... ✔
simple free choice — *all transitions sharing a common input place have no other input place* ..... ? [a]
extended free choice — *all transitions sharing a common input place have the same input places* ..... ? [b]
state machine — *every transition has exactly one input place and exactly one output place* ..... ✗ [c]
marked graph — *every place has exactly one input transition and exactly one output transition* ..... ✗ [d]
connected — *there is an undirected path between every two nodes (places or transitions)* ..... ? [e]
strongly connected — *there is a directed path between every two nodes (places or transitions)* ..... ✗ [f]
source place(s) — *one or more places have no input transitions* ..... ✔ [g]
sink place(s) — *one or more places have no output transitions* ..... ✔ [h]
source transition(s) — *one or more transitions have no input places* ..... ✗ [i]
sink transitions(s) — *one or more transitions have no output places* ..... ✗ [j]
loop-free — *no transition has an input place that is also an output place* ..... ✔ [k]
conservative — *for each transition, the number of input arcs equals the number of output arcs* ..... ✗ [l]
subconservative — *for each transition, the number of input arcs equals or exceeds the number of output arcs* ..... ✔ [m]

[a] stated by CÆSAR.BDD version 3.4 to be true on 2 instance(s) out of 20, and false on the remaining 18 instance(s).
[b] stated by CÆSAR.BDD version 3.4 to be true on 2 instance(s) out of 20, and false on the remaining 18 instance(s).
[c] stated by CÆSAR.BDD version 3.4 on all 20 instances (versions A and B, with N between 1 and 16).
[d] stated by CÆSAR.BDD version 3.4 on all 20 instances (versions A and B, with N between 1 and 16).
[e] stated by CÆSAR.BDD version 3.4 on all 20 instances (versions A and B, with N between 1 and 16).
[f] stated by CÆSAR.BDD version 3.4 on all 20 instances (versions A and B, with N between 1 and 16).
[g] stated by CÆSAR.BDD version 3.4 on all 20 instances (versions A and B, with N between 1 and 16).
[h] stated by CÆSAR.BDD version 3.4 on all 20 instances (versions A and B, with N between 1 and 16).
[i] stated by CÆSAR.BDD version 3.4 on all 20 instances (versions A and B, with N between 1 and 16).
[j] stated by CÆSAR.BDD version 3.4 on all 20 instances (versions A and B, with N between 1 and 16).
[k] stated by CÆSAR.BDD version 3.4 on all 20 instances (versions A and B, with N between 1 and 16).
[l] stated by CÆSAR.BDD version 3.4 on all 20 instances (versions A and B, with N between 1 and 16).
[m] stated by CÆSAR.BDD version 3.4 on all 20 instances (versions A and B, with N between 1 and 16).

Elvio G. Amparore
amparore@di.unito.it

nested units — *places are structured into hierarchically nested sequential units* [n] ..... ✔

Behavioural properties

safe — *in every reachable marking, there is no more than one token on a place* ..... ✔ [o]
dead place(s) — *one or more places have no token in any reachable marking* ..... ? [p]
dead transition(s) — *one or more transitions cannot fire from any reachable marking* ..... ? [q]
deadlock — *there exists a reachable marking from which no transition can be fired* ..... ✔ [r]
reversible — *from every reachable marking, there is a transition path going back to the initial marking* ..... ✘ [s]
live — *for every transition t, from every reachable marking, one can reach a marking in which t can fire* ..... ✘ [t]

Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
$N = 1$ (version A)	2	1	1	3
$N = 2$ (version A)	35	72	1	12
$N = 3$ (version A)	11,776	56,619	1	27
$N = 4$ (version A)	127,545,137	1,134,314,176	1	48
$N = 5$ (version A)	?	?	1	75 [u]
$N = 6$ (version A)	?	?	1	108 [v]
$N = 7$ (version A)	?	?	1	147 [w]
$N = 8$ (version A)	?	?	1	192 [x]
$N = 9$ (version A)	?	?	1	243 [y]
$N = 10$ (version A)	?	?	1	300 [z]
$N = 11$ (version A)	?	?	1	363 [aa]
$N = 12$ (version A)	?	?	1	432 [ab]
$N = 13$ (version A)	?	?	1	507 [ac]
$N = 14$ (version A)	?	?	1	588 [ad]
$N = 15$ (version A)	?	?	1	675 [ae]
$N = 16$ (version A)	?	?	1	768 [af]
$N = 1$ (version B)	2	1	1	4
$N = 4$ (version B)	61,556,225	526,297,216	1	64
$N = 9$ (version B)	?	?	1	324 [ag]
$N = 16$ (version B)	?	?	1	1024 [ah]

[n] the definition of Nested-Unit Petri Nets (NUPN) is available from http://mcc.lip6.fr/nupn.php
[o] by conception; confirmed by CÆSAR.BDD version 3.4 to be true on 5 instance(s) out of 20.
[p] stated by CÆSAR.BDD version 3.4 to be false on 6 instance(s) out of 20, and unknown on the remaining 14 instance(s).
[q] stated by CÆSAR.BDD version 3.4 to be false on 6 instance(s) out of 20, and unknown on the remaining 14 instance(s).
[r] stated by CÆSAR.BDD version 3.4 to be true on 5 instance(s) out of 20, and unknown on the remaining 15 instance(s).
[s] stated by CÆSAR.BDD version 3.4 to be false on 5 instance(s) out of 20, and unknown on the remaining 15 instance(s).
[t] stated by CÆSAR.BDD version 3.4 to be false on 5 instance(s) out of 20, and unknown on the remaining 15 instance(s).
[u] number of initial tokens, because the net is sub-conservative.
[v] number of initial tokens, because the net is sub-conservative.
[w] number of initial tokens, because the net is sub-conservative.
[x] number of initial tokens, because the net is sub-conservative.
[y] number of initial tokens, because the net is sub-conservative.
[z] number of initial tokens, because the net is sub-conservative.
[aa] number of initial tokens, because the net is sub-conservative.
[ab] number of initial tokens, because the net is sub-conservative.
[ac] number of initial tokens, because the net is sub-conservative.
[ad] number of initial tokens, because the net is sub-conservative.
[ae] number of initial tokens, because the net is sub-conservative.
[af] number of initial tokens, because the net is sub-conservative.
[ag] number of initial tokens, because the net is sub-conservative.
[ah] number of initial tokens, because the net is sub-conservative.