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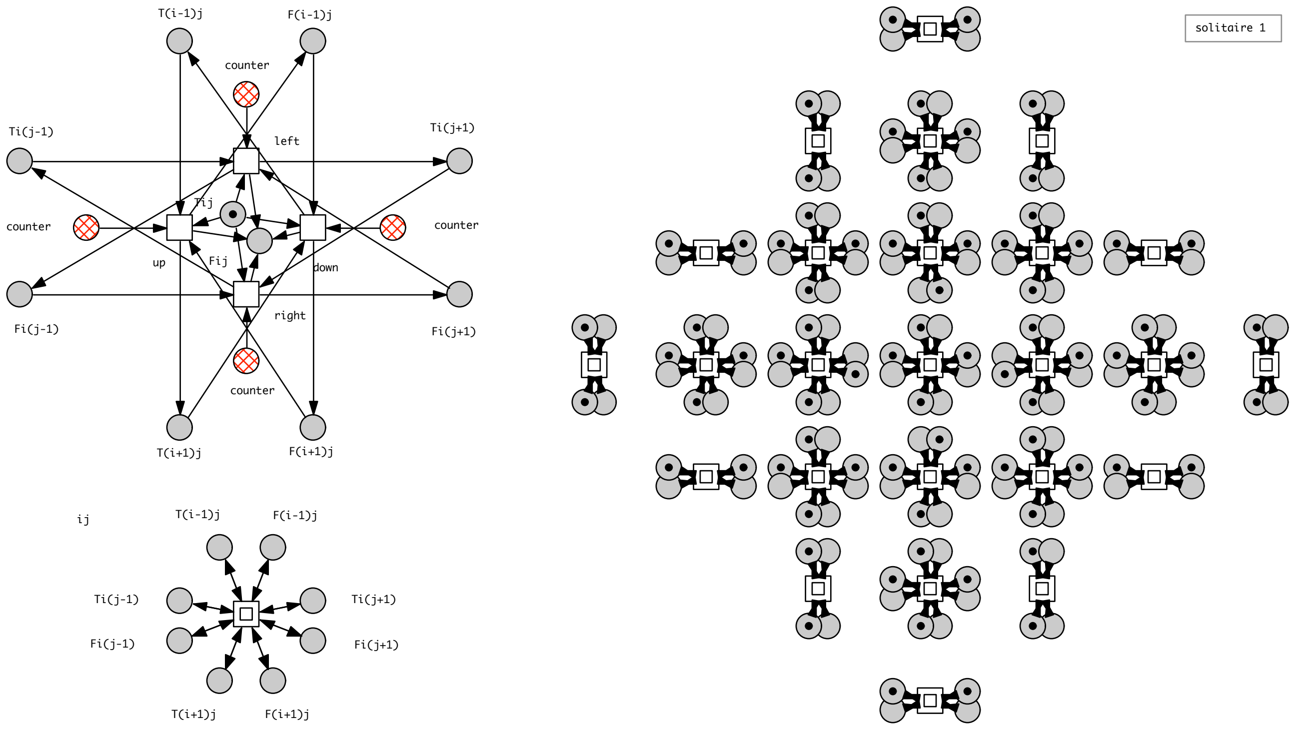
This form is a summary description of the model entitled "Solitaire" proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.

Description

Solitaire is a popular board game requiring non-obvious solution strategies; see [wiki] for the rules of the game. The objective of the Petri nets is to generate one/some/all strategies (paths) to reach a solution, i.e., a state where just one stone is left. The auxiliary place *counter* gives the current number of stones on the board; added to simplify the specification of the target state (any state with *counter* = 1). Solitaire is played on different boards; we give Petri nets for the most popular ones: square board (0), English board (1), French board (2), each in two versions: with/out counter [H05]. The existence of a solution may depend on the initially empty field; all initial markings have been chosen to enable a solution. Encoding this game as coloured Petri net would permit the generation of arbitrary boards of scalable size.

In March 2020, Pierre Bouvier and Hubert Garavel provided a decomposition of one instance of this model into a network of communicating automata. This network is expressed as a Nested-Unit Petri Net (NUPN) that can be found in the "toolspecific" section of the corresponding PNML file. In April 2021, Pierre Bouvier decomposed two more instances of this model.



General solitaire pattern for one field (left), and its composition to the 7×7 *English board (right).*

References

H05 M Heiner: About some Applications of Petri Net Theory - My Petri Net Picture Book; Talk, Adventmatik 2003, Paderborn, December 2003, http://www-dssz.informatik.tu-cottbus.de/publications/slides/2003_paderborn_pn_applications.sld.pdf.

Wiki Wikipedia: Peg solitaire; http://en.wikipedia.org/wiki/Peg_solitaire, last access 12/2013.

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Scaling parameter

Parameter name	Parameter description	Chosen parameter values
B	shape and size of the board	5×5 square board (0), 7×7 English board (1), 7×7 French board (3)

Size of the model

Parameter	Number of places	Number of transitions	Number of arcs	Number of units	HWB code
$B = 0$	50	84	456	26	1–25–25
$B = 0$, with counter	51	84	540	–	– – 51
$B = 1$	66	76	456	34	1–33–33
$B = 1$, with counter	67	76	532	–	– – 67
$B = 2$	74	92	552	38	1–37–37
$B = 2$, with counter	75	92	644	–	– – 75

Structural properties

ordinary — *all arcs have multiplicity one* ✔
simple free choice — *all transitions sharing a common input place have no other input place* ✗ [a]
extended free choice — *all transitions sharing a common input place have the same input places* ✗ [b]
state machine — *every transition has exactly one input place and exactly one output place* ✗ [c]
marked graph — *every place has exactly one input transition and exactly one output transition* ✗ [d]
connected — *there is an undirected path between every two nodes (places or transitions)* ✔ [e]
strongly connected — *there is a directed path between every two nodes (places or transitions)* ? [f]
source place(s) — *one or more places have no input transitions* ? [g]
sink place(s) — *one or more places have no output transitions* ✗ [h]
source transition(s) — *one or more transitions have no input places* ✗ [i]
sink transitions(s) — *one or more transitions have no output places* ✗ [j]
loop-free — *no transition has an input place that is also an output place* ✔ [k]
conservative — *for each transition, the number of input arcs equals the number of output arcs* ? [l]
subconservative — *for each transition, the number of input arcs equals or exceeds the number of output arcs* ✔ [m]
nested units — *places are structured into hierarchically nested sequential units* [n] ? [o]

Behavioural properties

safe — *in every reachable marking, there is no more than one token on a place* ? [p]
dead place(s) — *one or more places have no token in any reachable marking* ? [q]

[a] stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
[b] stated by CÆSAR.BDD version 2.6 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
[c] stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
[d] stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
[e] stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
[f] stated by CÆSAR.BDD version 2.0 to be false on all 3 instances with counters, and true on all 3 instances without counters.
[g] stated by CÆSAR.BDD version 2.0 to be true on all 3 instances with counters, and false on all 3 instances without counters.
[h] stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
[i] stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
[j] stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
[k] stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
[l] stated by CÆSAR.BDD version 2.0 to be false on all 3 instances with counters, and true on all 3 instances without counters.
[m] stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
[n] the definition of Nested-Unit Petri Nets (NUPN) is available from `http://mcc.lip6.fr/nupn.php`
[o] stated by CÆSAR.BDD version 3.5 to be false on all 3 instances with counter, and true on all 3 instances without counters.
[p] the nets corresponding to instances without counters are safe because they are covered with P-invariants having a single token in the initial place – found by CÆSAR.BDD version 3.3 to be false on all 3 instances with counter, and true on all 3 instances without counters.
[q] stated by CÆSAR.BDD version 3.3 to be false on 3 instance(s) out of 6, and unknown on the remaining 3 instance(s).

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dead transition(s) — *one or more transitions cannot fire from any reachable marking* .... ✗
deadlock — *there exists a reachable marking from which no transition can be fired* .... ✔ [r]
reversible — *from every reachable marking, there is a transition path going back to the initial marking* .... ✗
live — *for every transition t, from every reachable marking, one can reach a marking in which t can fire* .... ✗

Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
$B = 0$	1.6098×10^7 [s]	2.1396×10^8 [t]	1 [u]	25 [v]
$B = 0$, with counter	?	?	24	49 [w]
$B = 1$	$\geq$ 2.59564e+07 [x]	?	1 [y]	33 [z]
$B = 1$, with counter	?	?	32	65 [aa]
$B = 2$	$\geq$ 23014 [ab]	?	1 [ac]	37 [ad]
$B = 2$, with counter	?	?	36	73 [ae]

Other properties

Deadlocks (dead states) which correspond to a solution can be identified by: sum over all places $T_{i,j} = 1$, or counter=0. All places are covered by 1-P-invariants, except the counter place. All nets enjoy some symmetries.

(r) special deadlocks (dead states) correspond to the solutions we are looking for; confirmed at MCC'2014 by Lola and Tapaal on all 6 instances.
(s) computed at MCC'2014 by Marcie, PNMC, and PNXDD; exact value: 16,098,428.
(t) computed at MCC'2014 by Marcie; exact value: 213,958,152.
(u) computed at MCC'2014 by Marcie and PNMC; confirmed by CÆSAR.BDD version 3.5.
(v) number of initial tokens, because the net is sub-conservative.
(w) number of initial tokens, because the net is sub-conservative.
(x) stated by CÆSAR.BDD version 3.5.
(y) the instance is safe.
(z) number of initial tokens, because the net is conservative.
(aa) number of initial tokens, because the net is sub-conservative.
(ab) stated by CÆSAR.BDD version 3.5.
(ac) the instance is safe.
(ad) number of initial tokens, because the net is conservative.
(ae) number of initial tokens, because the net is sub-conservative.