

*This form is a summary description of the model entitled "NeighborGrid" proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.*

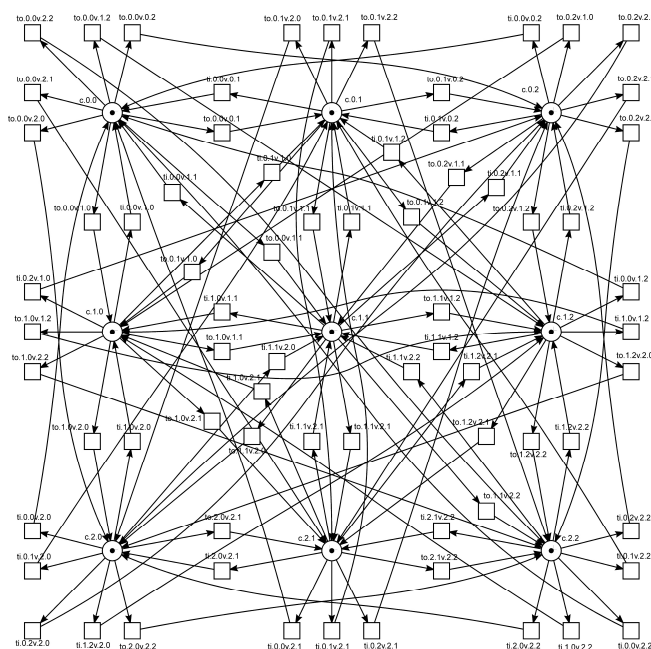
## Description

There is a  $d$ -dimensional grid of size  $n$  indexed with  $d$ -tuples having components' range from 0 to  $n - 1$ . A grid cell model is represented with a single Petri net place denoted as "p". Neighboring cells are connected via pairs of dedicated transitions; transitions are denoted as input "ti" and output "to" with respect to a cell with lesser index. A hypertorus is obtained from a hypercube via closing (connecting) opposite facets in each dimension. Indices are printed with '.' separator on dimensions; character 'v' separates two indices in a couple. More complicated cell models can be inserted but the canvas of connections does not change.

In a generalized neighborhood [1], neighbors are situated at Chebyshev distance equal to 1 restricted by a given interval of Manhattan distance  $r$ ,  $1 \leq r_1 \leq r \leq r_2 \leq d$ . Neighbors are connected via facets which are hypercubes having dimensions from  $d - r_1$  to  $d - r_2$ . Thus,  $r_1 = 1, r_2 = 1$  gives von-Neumann's neighborhood and  $r_1 = 1, r_2 = d$  gives Moore's neighborhood.

A program *hmn* [2] that generates models has the following command line: *hmn d n [m] [e] [r<sub>1</sub>] [r<sub>2</sub>] > hmn\_model.net*

where  $d$  is the number of dimensions ( $d \geq 1$ );  $n$  is the size of hypertorus or hypercube ( $n \geq 1$ ; for hypertorus  $n \geq 3$ ), actually the size is  $n \times n \times n \times \dots \times n$  ( $d$  times);  $m$  is the number of tokens in each node ( $m \geq 0$ , default 1);  $e$  is an edge condition: 't' – hypertorus, 'c' – hypercube (default 't');  $r_1$  is a lower bound of Manhattan distance (default  $r_1 = 1$ );  $r_2$  is an upper bound of Manhattan distance (default  $r_2 = d$ ),  $1 \leq r_1 \leq r_2 \leq d$ .



Graphical representation for  $d = 2, n = 3, m = 1, e = 't', r_1 = 1, r_2 = 2$

## References

- [1] Zaitsev D.A. A generalized neighborhood for cellular automata, Theoretical Computer Science. Online 22 November 2016, <http://dx.doi.org/10.1016/j.tcs.2016.11.002>
- [2] Zaitsev D.A. Generators of canvas for Petri net models of hypertorus (hypercube) grid with Moore's, von-Neumann's, and generalized neighborhoods, <https://github.com/dazeorgacm/hmn>

## Scaling parameter

Parameter name	Parameter description	Chosen parameter values
$d, n, m, e, r_1, r_2$	$d$ – dimension; $n$ – size; $m$ – initial marking of each place; $e$ – edge condition: 't' – hypertorus, 'c' – hypercube; $r_1$ – lower Manhattan distance; $r_2$ – upper Manhattan distance	(2,3,1,'t',1,2), (2,3,1,'c',1,2), (3,3,1,'t',1,1), (4,3,2,'c',2,3), (5,4,1,'t',3,5)

## Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
$N = (d, n, m, e, r_1, r_2)$	$P = n^d$	for a hypertorus ( $e='t'$ ): $T = n^d \cdot \sum_{j=r_1}^{r_2} 2^j C_d^j$ ; for a hypercube – sum on all the places (cells) the number of a cell neighbors (depending on the dimension of the corresponding edge)	$A = 2T$
(2,3,3,c,1,2)	9	40	80
(2,3,3,t,1,2)	9	72	144
(3,3,3,t,1,1)	27	162	324
(4,3,3,c,2,3)	81	1632	3264
(5,4,4,t,3,5)	1024	196608	393216

## Structural properties

- ordinary — all arcs have multiplicity one ..... ✓
- simple free choice — all transitions sharing a common input place have no other input place ..... ✓ (a)
- extended free choice — all transitions sharing a common input place have the same input places ..... ✓ (b)
- state machine — every transition has exactly one input place and exactly one output place ..... ✓ (c)
- marked graph — every place has exactly one input transition and exactly one output transition ..... ✗ (d)
- connected — there is an undirected path between every two nodes (places or transitions) ..... ✓ (e)
- strongly connected — there is a directed path between every two nodes (places or transitions) ..... ✓ (f)
- source place(s) — one or more places have no input transitions ..... ✗ (g)
- sink place(s) — one or more places have no output transitions ..... ✗ (h)
- source transition(s) — one or more transitions have no input places ..... ✗ (i)
- sink transitions(s) — one or more transitions have no output places ..... ✗ (j)
- loop-free — no transition has an input place that is also an output place ..... ✓ (k)
- conservative — for each transition, the number of input arcs equals the number of output arcs ..... ✓ (l)
- subconservative — for each transition, the number of input arcs equals or exceeds the number of output arcs ..... ✓ (m)
- nested units — places are structured into hierarchically nested sequential units<sup>(n)</sup> ..... ✗

(a) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).  
 (b) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).  
 (c) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).  
 (d) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).  
 (e) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).  
 (f) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).  
 (g) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).  
 (h) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).  
 (i) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).  
 (j) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).  
 (k) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).  
 (l) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).  
 (m) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).  
 (n) the definition of Nested-Unit Petri Nets (NUPN) is available from <http://mcc.lip6.fr/nupn.php>

## Behavioural properties

- safe** — *in every reachable marking, there is no more than one token on a place* ..... ✗<sup>(o)</sup>  
**dead place(s)** — *one or more places have no token in any reachable marking* ..... ✗<sup>(p)</sup>  
**dead transition(s)** — *one or more transitions cannot fire from any reachable marking* ..... ✗<sup>(q)</sup>  
**deadlock** — *there exists a reachable marking from which no transition can be fired* ..... ✗  
**reversible** — *from every reachable marking, there is a transition path going back to the initial marking* ..... ✓<sup>(r)</sup>  
**live** — *for every transition  $t$ , from every reachable marking, one can reach a marking in which  $t$  can fire* ..... ✓<sup>(s)</sup>

## Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
$d, n, m, e, r_1, r_2$	$C_{(m+1)n^d-1}^{n^d-1}$	Sum on all markings, for all places (cells) with nonzero marking, the number of the corresponding cell neighbors	$m \cdot n^d$	$m \cdot n^d$
(2,3,1,c,1,2)	24310	514800	9	9 <sup>(t)</sup>
(2,3,1,t,1,2)	24310	926640	9	9 <sup>(u)</sup>
(3,3,1,t,1,1)	973469712824056	?	27	27 <sup>(v)</sup>
(4,3,2,c,2,3)	$C_{242}^{80}$	?	162	162 <sup>(w)</sup>
(5,4,1,t,3,5)	$C_{2047}^{1023}$	?	1024	1024 <sup>(x)</sup>

<sup>(o)</sup> confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).  
<sup>(p)</sup> stated by [CÆSAR.BDD](#) version 3.3 on all 5 instances (see all aforementioned parameter values).  
<sup>(q)</sup> stated by [CÆSAR.BDD](#) version 2.7 to be false on 4 instances out of 5, and unknown on the remaining instance.  
<sup>(r)</sup> stated by [CÆSAR.BDD](#) version 2.7 to be true on 4 instances out of 5, and unknown on the remaining instance.  
<sup>(s)</sup> stated by [CÆSAR.BDD](#) version 2.7 to be true on 4 instances out of 5, and unknown on the remaining instance.  
<sup>(t)</sup> number of initial tokens, because the net is conservative.  
<sup>(u)</sup> number of initial tokens, because the net is conservative.  
<sup>(v)</sup> number of initial tokens, because the net is conservative.  
<sup>(w)</sup> number of initial tokens, because the net is conservative.  
<sup>(x)</sup> number of initial tokens, because the net is conservative.