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Description

Hypercube communication grid model [1,2] is composed of nodes which represent data communication equipment (DCE) implementing packet forwarding based on store-and-forward principle. Each DCE has ports, situated on facets of a unit size hypercube, which work in full-duplex mode. Data terminal equipment (DTE) is attached on the hypercube borders. Each DTE receives and sends packets.

Remind that, a d-dimension hypercube has $2 \cdot d$ facets each represents a $(d-1)$-dimension hypercube.

DCE index $(i_1, i_2, ..., i_d)$, where $1 \leq i_j \leq k$, $1 \leq j \leq d$, reflects its location within hypercube. Port index (r, n) consists of dimension number $1 \leq r \leq d$, a facet is perpendicular to, and direction number $1 \leq n \leq 2$, where $n = 1$ represents the direction to the origin of coordinates and $n = 2$ represents the direction to infinity.

DCE model contains an internal buffer represented with $2 \cdot d + 1$ places: the avaliable buffer size and buffer sections for storing packets forwarded to the corresponding ports.

Each of $2 \cdot d$ DCE ports has two tracts: input and output. Memory of a tract is represented with two places – the tract buffer and the tract buffer available capacity (usually equat to unit). An output tract work is modeled by a single transition taking a packet from the corresponding section of the internal buffer and putting it into the tract buffer. An input tract work is modeled by $2 \cdot d - 1$ transitions forwarding arrived packet from the tract buffer to the corresponding section of the internal buffer except of the arrival port number.

A hypercube is composed via meging tract places of neighbor DCE which has a common facet: input tract of one DCE with output tract of the other DCE and vice versa.

On the borders, which constitute $2 \cdot d$ hypercubes of dimension $d - 1$, DTE models are attached. A simple DTE model is represented with a single transition that receives a packet from a neighbor DCE output tract and sends a packet into the neighbor DCE input tract.

For planar case when $d = 2$, models are described in [1,3] with simplified notation of ports.

References

[1] Zaitsev D.A., Zaitsev I.D., Shmeleva T.R. Infinite Petri Nets as Models of Grids (pp. 187-204). Chapter 19 in Mehdi Khosrow-Pour (Ed.) Encyclopedia of Information Science and Technology, Third Edition (10 Volumes). IGI-Global: USA, 2014.

[2] Zaitsev D.A., Shmeleva T.R. Hypercube communication structures analysis via parametric Petri nets. Proceedings of 24th UK Performance Engineering Workshop (UKPEW 2008), 3-4 July 2008, Department of Computing, Imperial College London, p. 358-371.

[3] Shmeleva T.R., Zaitsev D.A., Zaitsev I.D. Analysis of Square Communication Grids via Infinite Petri Nets. Transactions of Odessa National Academy of Telecommunication, no. 1, 2009, p. 27-35.

[4] A C program that generates k^d hypercube can be downloaded from http://daze.ho.ua/tinaz.zip

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Scaling parameter

Parameter name	Parameter description	Chosen parameter values
d, k, p, b	d is the nummber of dimensions; k is the hypercube size of k^d DCE nodes and $2 \cdot d \cdot k^{d-1}$ DTE nodes; p is the number of packets in each section of internal buffer; b is the available size of internal buffer; p and b define the initial marking and do not affect the model structure.	$(d, k) = (3, 4), (4, 3), (5, 3)$ with $p = k$ and $b = d \cdot k$

Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
(d, k)	$P = 6 \cdot d \cdot k^d + k^d + 4 \cdot d \cdot k^{d-1}$	$T = 4 \cdot d^2 \cdot k^d + 2 \cdot d \cdot k^{d-1}$	$A = 16 \cdot d^2 \cdot k^d + 8 \cdot d \cdot k^{d-1}$
$(d = 3, k = 4)$	1408	2400	9600
$(d = 4, k = 3)$	2457	5400	21600
$(d = 5, k = 3)$	9153	25110	100440

Structural properties

ordinary — *all arcs have multiplicity one* ✔
simple free choice — *all transitions sharing a common input place have no other input place* ✗ [(a)]
extended free choice — *all transitions sharing a common input place have the same input places* ✗ [(b)]
state machine — *every transition has exactly one input place and exactly one output place* ✗ [(c)]
marked graph — *every place has exactly one input transition and exactly one output transition* ✗ [(d)]
connected — *there is an undirected path between every two nodes (places or transitions)* ✔ [(e)]
strongly connected — *there is a directed path between every two nodes (places or transitions)* ✔ [(f)]
source place(s) — *one or more places have no input transitions* ✗ [(g)]
sink place(s) — *one or more places have no output transitions* ✗ [(h)]
source transition(s) — *one or more transitions have no input places* ✗ [(i)]
sink transitions(s) — *one or more transitions have no output places* ✗ [(j)]
loop-free — *no transition has an input place that is also an output place* ✔ [(k)]
conservative — *for each transition, the number of input arcs equals the number of output arcs* ✔ [(l)]
subconservative — *for each transition, the number of input arcs equals or exceeds the number of output arcs* ✔ [(m)]
nested units — *places are structured into hierarchically nested sequential units* [(n)] ✗

[(a)] stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
[(b)] stated by CÆSAR.BDD version 2.6 on all 3 instances ((3, 4), (4, 3), (5, 3)).
[(c)] stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
[(d)] stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
[(e)] stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
[(f)] stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
[(g)] stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
[(h)] stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
[(i)] stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
[(j)] stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
[(k)] stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
[(l)] stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
[(m)] stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
[(n)] the definition of Nested-Unit Petri Nets (NUPN) is available from `http://mcc.lip6.fr/nupn.php`

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Behavioural properties

safe — *in every reachable marking, there is no more than one token on a place* .... ✗ (o)
dead place(s) — *one or more places have no token in any reachable marking* .... ?
dead transition(s) — *one or more transitions cannot fire from any reachable marking* .... ✗
deadlock — *there exists a reachable marking from which no transition can be fired* .... ✗ (p)
reversible — *from every reachable marking, there is a transition path going back to the initial marking* .... ✗
live — *for every transition t, from every reachable marking, one can reach a marking in which t can fire* .... ✗

Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
$(d = 3, k = 4)$	?	?	?	2784 (q)
$(d = 4, k = 3)$	?	?	?	3780 (r)
$(d = 5, k = 3)$	?	?	?	14175 (s)

Other properties

Model is $2 \cdot d \cdot p + b$ bounded — the sum of tokens in DCE internal buffer places. Model is P/T-invariant for any natural k as proven in [1,2]

(o) stated by CÆSAR.BDD version 2.2 on all 3 instances $((3, 4), (4, 3), (5, 3))$.
(p) proven in [1,2]; checked by the Tina http://www.laas.fr/tina tool version 3.3.0 as well as other behavioural properties for small values of parameters d, k.
(q) number of initial tokens, because the net is conservative.
(r) number of initial tokens, because the net is conservative.
(s) number of initial tokens, because the net is conservative.