Dmitry Zaitsev
daze@acm.org

This form is a summary description of the model entitled "NeighborGrid" proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.

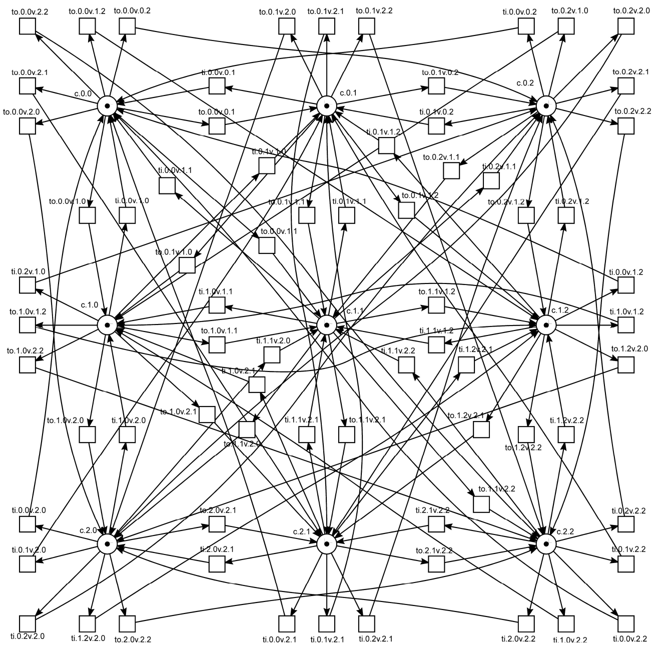
Description

There is a d-dimensional grid of size n indexed with d-tuples having components' range from 0 to $n-1$. A grid cell model is represented with a single Petri net place denoted as "p". Neighboring cells are connected via pairs of dedicated transitions; transitions are denoted as input "ti" and output "to" with respect to a cell with lesser index. A hypertorus is obtained from a hypercube via closing (connecting) opposite facets in each dimension. Indices are printed with '.' separator on dimensions; character 'v' separates two indices in a couple. More complicated cell models can be inserted but the canvas of connections does not change.

In a generalized neighborhood [1], neighbors are situated at Chebyshev distance equal to 1 restricted by a given interval of Manhattan distance r, $1 \le r_1 \le r \le r_2 \le d$. Neighbors are connected via facets which are hypercubes having dimensions from $d-r_1$ to $d-r_2$. Thus, $r_1 = 1$, $r_2 = 1$ gives von-Neumann's neighborhood and $r_1 = 1$, $r_2 = d$ gives Moore's neighborhood.

A program *hmn* [2] that generates models has the following command line: *hmn d n* [*m*] [*e*] [r_1] [r_2] > *hmn_model.net*

where d is the number of dimensions ($d \ge 1$); n is the size of hypertorus or hypercube ($n \ge 1$; for hypertorus $n \ge 3$), actually the size is $n \times n \times n \times ... \times n$ (d times); m is the number of tokens in each node ($m \ge 0$, default 1); e is an edge condition: 't' – hypertorus, 'c' – hypercube (default 't'); r_1 is a lower bound of Manhattan distance (default $r_1 = 1$); r_2 is an upper bound of Manhattan distance (default $r_2 = d$), $1 \le r_1 \le r_2 \le d$.



Graphical representation for $d = 2$, $n = 3$, $m = 1$, $e = ' t'$, $r_1 = 1$, $r_2 = 2$

References

[1] Zaitsev D.A. A generalized neighborhood for cellular automata, Theoretical Computer Science. Online 22 November 2016, http://dx.doi.org/10.1016/j.tcs.2016.11.002

[2] Zaitsev D.A. Generators of canvas for Petri net models of hypertorus (hypercube) grid with Moore's, von-Neumann's, and generalized neighborhoods, https://github.com/dazeorgacm/hmn

Dmitry Zaitsev
daze@acm.org

Scaling parameter

Parameter name	Parameter description	Chosen parameter values
d, n, m, e, r_1, r_2	d – dimension; n – size; m – initial marking of each place; e – edge condition: 't' – hypertorus, 'c' – hypercube; r_1 – lower Manhattan distance; r_2 – upper Manhattan distance	(2,3,1,'t',1,2), (2,3,1,'c',1,2), (3,3,1,'t',1,1), (4,3,2,'c',2,3), (5,4,1,'t',3,5)

Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
$N = (d, n, m, e, r_1, r_2)$	$P = n^d$	for a hypertorus (e='t'): $T = n^d \cdot \sum_{j=r_1}^{r_2} 2^j C_d^j$; for a hypercube – sum on all the places (cells) the number of a cell neighbors (depending on the dimension of the corresponding edge)	$A = 2T$
(2,3,3,c,1,2)	9	40	80
(2,3,3,t,1,2)	9	72	144
(3,3,3,t,1,1)	27	162	324
(4,3,3,c,2,3)	81	1632	3264
(5,4,4,t,3,5)	1024	196608	393216

Structural properties

ordinary — *all arcs have multiplicity one* ✔
simple free choice — *all transitions sharing a common input place have no other input place* ✔ (a)
extended free choice — *all transitions sharing a common input place have the same input places* ✔ (b)
state machine — *every transition has exactly one input place and exactly one output place* ✔ (c)
marked graph — *every place has exactly one input transition and exactly one output transition* ✗ (d)
connected — *there is an undirected path between every two nodes (places or transitions)* ✔ (e)
strongly connected — *there is a directed path between every two nodes (places or transitions)* ✔ (f)
source place(s) — *one or more places have no input transitions* ✗ (g)
sink place(s) — *one or more places have no output transitions* ✗ (h)
source transition(s) — *one or more transitions have no input places* ✗ (i)
sink transitions(s) — *one or more transitions have no output places* ✗ (j)
loop-free — *no transition has an input place that is also an output place* ✔ (k)
conservative — *for each transition, the number of input arcs equals the number of output arcs* ✔ (l)
subconservative — *for each transition, the number of input arcs equals or exceeds the number of output arcs* ✔ (m)
nested units — *places are structured into hierarchically nested sequential units* (n) ✗

(a) confirmed by CÆSAR.BDD version 2.7 on all 5 instances (see all aforementioned parameter values).
(b) confirmed by CÆSAR.BDD version 2.7 on all 5 instances (see all aforementioned parameter values).
(c) confirmed by CÆSAR.BDD version 2.7 on all 5 instances (see all aforementioned parameter values).
(d) confirmed by CÆSAR.BDD version 2.7 on all 5 instances (see all aforementioned parameter values).
(e) confirmed by CÆSAR.BDD version 2.7 on all 5 instances (see all aforementioned parameter values).
(f) confirmed by CÆSAR.BDD version 2.7 on all 5 instances (see all aforementioned parameter values).
(g) confirmed by CÆSAR.BDD version 2.7 on all 5 instances (see all aforementioned parameter values).
(h) confirmed by CÆSAR.BDD version 2.7 on all 5 instances (see all aforementioned parameter values).
(i) confirmed by CÆSAR.BDD version 2.7 on all 5 instances (see all aforementioned parameter values).
(j) confirmed by CÆSAR.BDD version 2.7 on all 5 instances (see all aforementioned parameter values).
(k) confirmed by CÆSAR.BDD version 2.7 on all 5 instances (see all aforementioned parameter values).
(l) confirmed by CÆSAR.BDD version 2.7 on all 5 instances (see all aforementioned parameter values).
(m) confirmed by CÆSAR.BDD version 2.7 on all 5 instances (see all aforementioned parameter values).
(n) the definition of Nested-Unit Petri Nets (NUPN) is available from http://mcc.lip6.fr/nupn.php

Dmitry Zaitsev
daze@acm.org

Behavioural properties

safe — *in every reachable marking, there is no more than one token on a place* .... ✗ [(o)]
deadlock — *there exists a reachable marking from which no transition can be fired* .... ✗
reversible — *from every reachable marking, there is a transition path going back to the initial marking* .... ✔ [(p)]
quasi-live — *for every transition t, there exists a reachable marking in which t can fire* .... ✔ [(q)]
live — *for every transition t, from every reachable marking, one can reach a marking in which t can fire* .... ✔ [(r)]

Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
d, n, m, e, r_1, r_2	$C^{n^d-1}_{(m+1)n^d-1}$	Sum on all markings, for all places (cells) with nonzero marking, the number of the corresponding cell neighbors	$m \cdot n^d$	$m \cdot n^d$
(2,3,1,c,1,2)	24310	514800	9	9 [(s)]
(2,3,1,t,1,2)	24310	926640	9	9 [(t)]
(3,3,1,t,1,1)	973469712824056	?	27	27 [(u)]
(4,3,2,c,2,3)	C^{80}_{242}	?	162	162 [(v)]
(5,4,1,t,3,5)	C^{1023}_{2047}	?	1024	1024 [(w)]

(o) confirmed by CÆSAR.BDD version 2.7 on all 5 instances (see all aforementioned parameter values).
(p) stated by CÆSAR.BDD version 2.7 to be true on 4 instances out of 5, and unknown on the remaining instance.
(q) stated by CÆSAR.BDD version 2.7 to be true on 4 instances out of 5, and unknown on the remaining instance.
(r) stated by CÆSAR.BDD version 2.7 to be true on 4 instances out of 5, and unknown on the remaining instance.
(s) number of initial tokens, because the net is conservative.
(t) number of initial tokens, because the net is conservative.
(u) number of initial tokens, because the net is conservative.
(v) number of initial tokens, because the net is conservative.
(w) number of initial tokens, because the net is conservative.