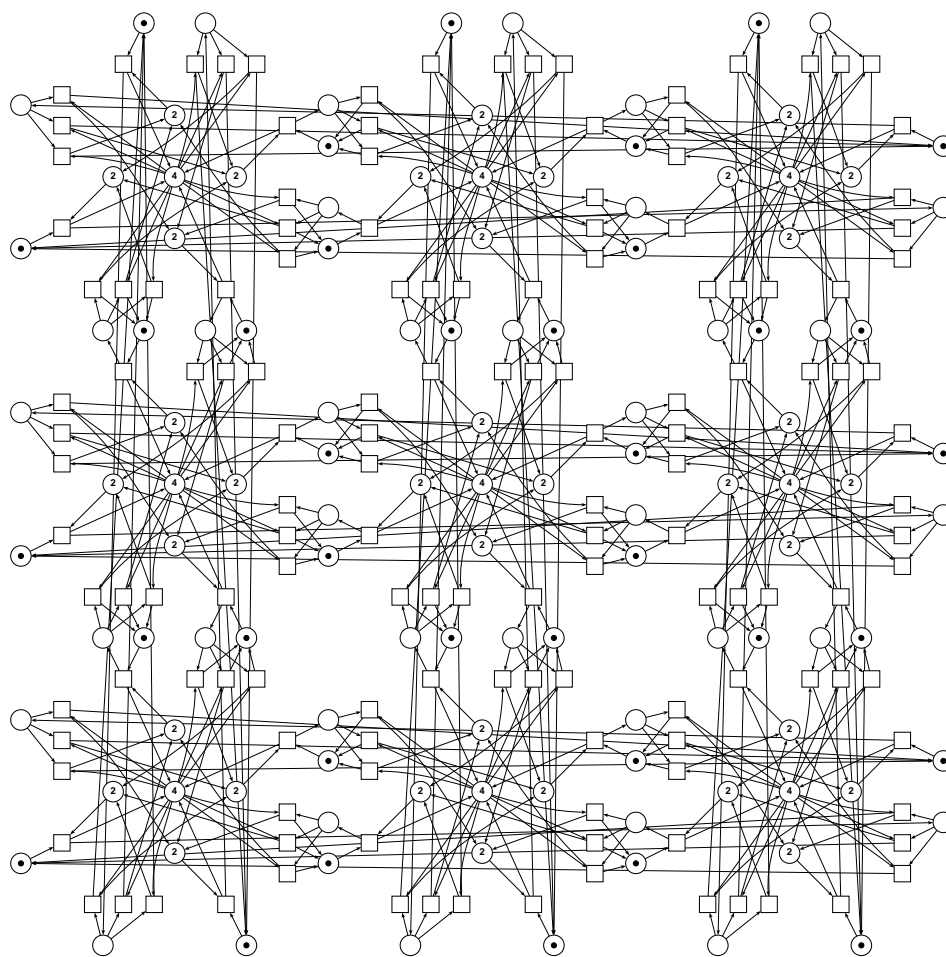


*This form is a summary description of the model entitled “HypertorusGrid” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.*

## Description

A hypertorus is obtained from a hypercube via closing (connecting) opposite facets in each dimension. A cell of hypertorus grid represents a computing and packet switching device with ports situated on facets of the unit-sized hypercube. A device works in full-duplex mode using store-and-forward principle with limited capacity of buffer. Neighboring cells are connected via merging contact places situated on common facets.



*Graphical representation for  $d = 2$ ,  $k = 3$ ,  $p = 2$ , and  $b = 4$ .*

## References

- [1] Zaitsev D.A., Zaitsev I.D., Shmeleva T.R. [Infinite Petri Nets as Models of Grids](#). Chapter 19 in Mehdi Khosrow-Pour (Ed.) Encyclopedia of Information Science and Technology, Third Edition (10 Volumes). IGI-Global: USA, pp. 187–204, 2014.
- [2] Zaitsev D.A. [Verification of Computing Grids with Special Edge Conditions by Infinite Petri Nets](#), Automatic Control and Computer Sciences, 2013, vol. 47, no. 7, pp.403–412.
- [3] Zaitsev D.A. Generator of hypertorus Petri net models. <http://github.com/dazeorgacm/htgen>

## Scaling parameter

Parameter name	Parameter description	Chosen parameter values
$(d, k, p, b)$	$d$ – dimension of hypertorus; $k$ – size in each dimension, totally $k^d$ nodes; $p$ – number of packets in each section of the internal buffer; $b$ – available buffer size. $d$ and $k$ influence the Petri net structure while $p$ and $b$ define its initial marking.	$(2,1,8,0)$ , $(2,2,1,0)$ , $(2,3,2,4)$ , $(3,3,2,6)$ , $(4,3,2,8)$ , $(5,3,2,10)$

## Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
$(d, k, p, b)$	$6 \cdot d \cdot k^d + k^d$	$4 \cdot d^2 \cdot k^d$	$16 \cdot d^2 \cdot k^d$
$(2, 1, 8, 0)$	13	16	64
$(2, 2, 1, 0)$	52	64	256
$(2, 3, 2, 4)$	117	144	576
$(3, 3, 2, 6)$	513	972	3888
$(4, 3, 2, 8)$	2025	5184	20736
$(5, 3, 2, 10)$	7533	24300	97200

## Structural properties

<b>ordinary</b> — all arcs have multiplicity one .....	✓
<b>simple free choice</b> — all transitions sharing a common input place have no other input place .....	✗ (a)
<b>extended free choice</b> — all transitions sharing a common input place have the same input places .....	✗ (b)
<b>state machine</b> — every transition has exactly one input place and exactly one output place .....	✗ (c)
<b>marked graph</b> — every place has exactly one input transition and exactly one output transition .....	✗ (d)
<b>connected</b> — there is an undirected path between every two nodes (places or transitions) .....	✓ (e)
<b>strongly connected</b> — there is a directed path between every two nodes (places or transitions) .....	✓ (f)
<b>source place(s)</b> — one or more places have no input transitions .....	✗ (g)
<b>sink place(s)</b> — one or more places have no output transitions .....	✗ (h)
<b>source transition(s)</b> — one or more transitions have no input places .....	✗ (i)
<b>sink transitions(s)</b> — one or more transitions have no output places .....	✗ (j)
<b>loop-free</b> — no transition has an input place that is also an output place .....	✓ (k)
<b>conservative</b> — for each transition, the number of input arcs equals the number of output arcs .....	✓ (l)
<b>subconservative</b> — for each transition, the number of input arcs equals or exceeds the number of output arcs .....	✓ (m)
<b>nested units</b> — places are structured into hierarchically nested sequential units <sup>(n)</sup> .....	✗

(a) stated by [CÆSAR.BDD](#) version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(b) stated by [CÆSAR.BDD](#) version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(c) stated by [CÆSAR.BDD](#) version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(d) stated by [CÆSAR.BDD](#) version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(e) stated by [CÆSAR.BDD](#) version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(f) stated by [CÆSAR.BDD](#) version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(g) stated by [CÆSAR.BDD](#) version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(h) stated by [CÆSAR.BDD](#) version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(i) stated by [CÆSAR.BDD](#) version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(j) stated by [CÆSAR.BDD](#) version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(k) stated by [CÆSAR.BDD](#) version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(l) stated by [CÆSAR.BDD](#) version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(m) stated by [CÆSAR.BDD](#) version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(n) the definition of Nested-Unit Petri Nets (NUPN) is available from <http://mcc.lip6.fr/nupn.php>

## Behavioural properties

- safe** — *in every reachable marking, there is no more than one token on a place* ..... X <sup>(o)</sup>  
**deadlock** — *there exists a reachable marking from which no transition can be fired* ..... ✓  
**reversible** — *from every reachable marking, there is a transition path going back to the initial marking* ..... X  
**quasi-live** — *for every transition  $t$ , there exists a reachable marking in which  $t$  can fire* ..... ✓ <sup>(p)</sup>  
**live** — *for every transition  $t$ , from every reachable marking, one can reach a marking in which  $t$  can fire* ..... X

## Size of the marking graphs

Parameter	Number of reach-able markings	Number of tran-sition firings	Max. number of tokens per place	Max. number of tokens per marking
(2, 1, 8, 0)	87552	667632	32	36
(2, 2, 1, 0)	$\geq 7.50898e+08$ <sup>(q)</sup>	?	?	$\geq 32$ <sup>(r)</sup>
(2, 3, 2, 4)	?	?	?	144 <sup>(s)</sup>
(3, 3, 2, 6)	?	?	?	648 <sup>(t)</sup>
(4, 3, 2, 8)	?	?	?	2592 <sup>(u)</sup>
(5, 3, 2, 10)	?	?	?	9720 <sup>(v)</sup>

## Other properties

To observe a deadlock, there should be enough packets ( $p$ ) compared to available buffer size ( $b$ ) to block a couple or more devices [1,2].

Using *htgen* [3], a model in format .net of Tina modeling system (<http://www.laas.fr/tina>) was built for given values of parameters.

<sup>(o)</sup> stated by CÆSAR.BDD version 2.6 on all 6 instances (i.e., the six quadruples listed above).

<sup>(p)</sup> stated by CÆSAR.BDD version 2.6 to be true on 2 instance(s) out of 6, and unknown on the remaining 4 instance(s).

<sup>(q)</sup> stated by CÆSAR.BDD version 2.6.

<sup>(r)</sup> lower bound given by the number of initial tokens.

<sup>(s)</sup> number of initial tokens, because the net is conservative.

<sup>(t)</sup> number of initial tokens, because the net is conservative.

<sup>(u)</sup> number of initial tokens, because the net is conservative.

<sup>(v)</sup> number of initial tokens, because the net is conservative.