Model: DoubleExponent
Type: PT
Origin: Academic

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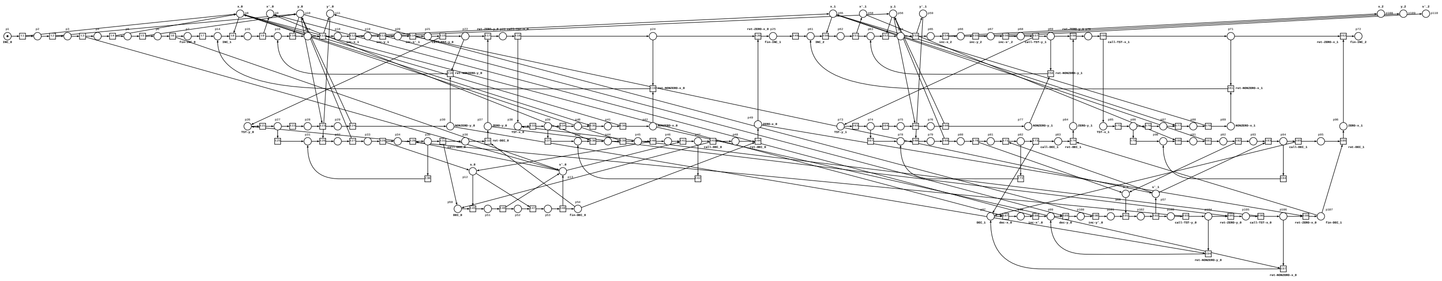
This form is a summary description of the model entitled "DoubleExponent" proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.

Description

The model illustrates difference beween weak, strong, and exact Petri net computers [1]. The generated for parameter k net [2] counts 2^{2^k} as a strong computer after Richard J. Lipton [3]. Well-structured net programs of Javier Esparza [4] have been encoded by Petri nets. The following marking belongs to the reachability space: $finINC_k = 1$; $x_0 = 2$, $y_0 = 2$, $s'_0 = 2$; $x_1 = 4$, $y_1 = 4$, $s'_1 = 4$; ...; $x_k = 2^{2^k}$, $y_k = 2^{2^k}$, $s'_k = 2^{2^k}$. It is the only marking with $finINC_k = 1$. For a given k, the resulting net is composed of a single $Block_0$ and repeated $(k-1)$ times $Block_i$. $Block_i$ includes subnets INC_{i+1}, $TSTy_i$, $TSTx_i$, DEC_i and variables (places) x_i, x'_i, y_i, y'_i, s_i, s'_i. Connections of neighboring blocks reflect initialization of the next variables x_{i+1}, y_{i+1}, s'_{i+1} and recursive calls of the previous block's $TSTy_{i-1}$, $TSTx_{i-1}$. Models are generated by program [2] which implements technique described in [5].



Graphical representation for $k = 2$

References

[1] Dmitry Zaitsev, Some Remarks on Petri Net Computers: Weak, Exact, and Strong. Petri Net Newsletter, Volume 85, December 2016, 3-7.

[2] Dmitry Zaitsev, Generator of Petri nets which count double exponent 2^{2^k} after R.J.Lipton and J.Esparza constructs. https://github.com/dazeorgacm/depn

[3] Richard J. Lipton, The Reachability Problem Requires Exponential Space, Technical Report 63, Yale University, 1976. http://www.cs.yale.edu/publications/techreports/tr63.pdf

[4] Javier Esparza, Decidability and Complexity of Petri Net Problems - An Introduction. LNCS 1491, 1996, 374-428. http://www7.in.tum.de/um/bibdb/esparza/course.pdf

[5] Dmitry A. Zaitsev, Ivan D. Zaitsev and Tatiana R. Shmeleva. Infinite Petri Nets: Part 2, Modeling Triangular, Hexagonal, Hypercube and Hypertorus Structures, Complex Systems, 26(4), 2017, 341-371.

Scaling parameter

Parameter name	Parameter description	Chosen parameter values
k	k – power of double exponent 2^{2^k} to compute	1,2,3,4,10,20,100,200

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Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
k	$P = 53(k-1) + 57$	$T = 50(k-1) + 48$	$A = 141(k-1) + 135$
$k = 1$	57	48	135
$k = 2$	110	98	276
$k = 3$	163	148	417
$k = 4$	216	198	558
$k = 10$	534	498	1404
$k = 20$	1064	998	2814
$k = 100$	5304	4998	14094
$k = 200$	10604	9998	28194

Structural properties

ordinary — *all arcs have multiplicity one* ✔
simple free choice — *all transitions sharing a common input place have no other input place* ✗ [a]
extended free choice — *all transitions sharing a common input place have the same input places* ✗ [b]
state machine — *every transition has exactly one input place and exactly one output place* ✗ [c]
marked graph — *every place has exactly one input transition and exactly one output transition* ✗ [d]
connected — *there is an undirected path between every two nodes (places or transitions)* ✔ [e]
strongly connected — *there is a directed path between every two nodes (places or transitions)* ✗ [f]
source place(s) — *one or more places have no input transitions* ✔ [g]
sink place(s) — *one or more places have no output transitions* ✔ [h]
source transition(s) — *one or more transitions have no input places* ✗ [i]
sink transitions(s) — *one or more transitions have no output places* ✗ [j]
loop-free — *no transition has an input place that is also an output place* ✔ [k]
conservative — *for each transition, the number of input arcs equals the number of output arcs* ✗ [l]
subconservative — *for each transition, the number of input arcs equals or exceeds the number of output arcs* ✗ [m]
nested units — *places are structured into hierarchically nested sequential units* [n] ✗

Behavioural properties

safe — *in every reachable marking, there is no more than one token on a place* ✗ [o]
deadlock — *there exists a reachable marking from which no transition can be fired* ✔
reversible — *from every reachable marking, there is a transition path going back to the initial marking* ✗
quasi-live — *for every transition t, there exists a reachable marking in which t can fire* ✔
live — *for every transition t, from every reachable marking, one can reach a marking in which t can fire* ✗

[a] stated by CÆSAR.BDD version 2.7 on all 8 instances (for k =1, 2, 3, 4, 10, 20, 100, and 200).
[b] stated by CÆSAR.BDD version 2.7 on all 8 instances (for k =1, 2, 3, 4, 10, 20, 100, and 200).
[c] stated by CÆSAR.BDD version 2.7 on all 8 instances (for k =1, 2, 3, 4, 10, 20, 100, and 200).
[d] stated by CÆSAR.BDD version 2.7 on all 8 instances (for k =1, 2, 3, 4, 10, 20, 100, and 200).
[e] stated by CÆSAR.BDD version 2.7 on all 8 instances (for k =1, 2, 3, 4, 10, 20, 100, and 200).
[f] stated by CÆSAR.BDD version 2.7 on all 8 instances (for k =1, 2, 3, 4, 10, 20, 100, and 200).
[g] stated by CÆSAR.BDD version 2.7 on all 8 instances (for k =1, 2, 3, 4, 10, 20, 100, and 200).
[h] stated by CÆSAR.BDD version 2.7 on all 8 instances (for k =1, 2, 3, 4, 10, 20, 100, and 200).
[i] stated by CÆSAR.BDD version 2.7 on all 8 instances (for k =1, 2, 3, 4, 10, 20, 100, and 200).
[j] stated by CÆSAR.BDD version 2.7 on all 8 instances (for k =1, 2, 3, 4, 10, 20, 100, and 200).
[k] stated by CÆSAR.BDD version 2.7 on all 8 instances (for k =1, 2, 3, 4, 10, 20, 100, and 200).
[l] stated by CÆSAR.BDD version 2.7 on all 8 instances (for k =1, 2, 3, 4, 10, 20, 100, and 200).
[m] stated by CÆSAR.BDD version 2.7 on all 8 instances (for k =1, 2, 3, 4, 10, 20, 100, and 200).
[n] the definition of Nested-Unit Petri Nets (NUPN) is available from `http://mcc.lip6.fr/nupn.php`
[o] stated by CÆSAR.BDD version 2.7 on all 8 instances (for k =1, 2, 3, 4, 10, 20, 100, and 200).

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Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
k	?	?	2^{2^k}	$> F$, $F = 3 \cdot \sum_{i=0}^{k} 2^{2^i} + 1$ (guess: $F + 2k$)
1	149	148	4	21
2	3708	3707	16	71
3	2,385,072	2,385,071	256	841
4	?	?	65,536	?
10	?	?	$\approx 1.797693135 \times 10^{308}$	?
20	?	?	$\sim 10^{314,573}$	?
100	?	?	$\sim 10^{38 \cdot 10^{28}}$	?
200	?	?	$\sim 10^{482 \cdot 10^{57}}$	?

Other properties

The net is a Strong Computer of 2^{2^k} and its size is linear in k that proves exponential complexity in space of Petri net reachability problem.