This form is a summary description of the model entitled “HypercubeCommunicationGrid” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.

Description

Hypercube communication grid model [1,2] is composed of nodes which represent data communication equipment (DCE) implementing packet forwarding based on store-and-forward principle. Each DCE has ports, situated on facets of a unit size hypercube, which work in full-duplex mode. Data terminal equipment (DTE) is attached on the hypercube borders. Each DTE receives and sends packets.

Remind that, a \( d \)-dimension hypercube has \( 2 \cdot d \) facets each represents a \((d - 1)\)-dimension hypercube.

DCE index \((i_1, i_2, ..., i_d)\), where \( 1 \leq i_j \leq k \), \( 1 \leq j \leq d \), reflects its location within hypercube. Port index \((r, n)\) consists of dimension number \( 1 \leq r \leq d \), a facet is perpendicular to, and direction number \( 1 \leq n \leq 2 \), where \( n = 1 \) represents the direction to the origin of coordinates and \( n = 2 \) represents the direction to infinity.

DCE model contains an internal buffer represented with \( 2 \cdot d + 1 \) places: the available buffer size and buffer sections for storing packets forwarded to the corresponding ports.

Each of \( 2 \cdot d \) DCE ports has two tracts: input and output. Memory of a tract is represented with two places – the tract buffer and the tract buffer available capacity (usually equal to unit). An output tract work is modeled by a single transition taking a packet from the corresponding section of the internal buffer and putting it into the tract buffer. An input tract work is modeled by \( 2 \cdot d - 1 \) transitions forwarding arrived packet from the tract buffer to the corresponding section of the internal buffer except of the arrival port number.

A hypercube is composed via merging tract places of neighbor DCE which has a common facet: input tract of one DCE with output tract of the other DCE and vice versa.

On the borders, which constitute \( 2 \cdot d \) hypercubes of dimension \( d - 1 \), DTE models are attached. A simple DTE model is represented with a single transition that receives a packet from a neighbor DCE output tract and sends a packet into the neighbor DCE input tract.

For planar case when \( d = 2 \), models are described in [1,3] with simplified notation of ports.

References


[4] A C program that generates \( k^d \) hypercube can be downloaded from http://daze.ho.ua/tinaz.zip
Scaling parameter

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Parameter description</th>
<th>Chosen parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d, k, p, b$</td>
<td>$d$ is the number of dimensions; $k$ is the hypercube size of $k^d$ DCE nodes and $2 \cdot d \cdot k^{d-1}$ DTE nodes; $p$ is the number of packets in each section of internal buffer; $b$ is the available size of internal buffer; $p$ and $b$ define the initial marking and do not affect the model structure.</td>
<td>$(d, k) = (3, 4), (4, 3), (5, 3)$ with $p = k$ and $b = d \cdot k$</td>
</tr>
</tbody>
</table>

Size of the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of places</th>
<th>Number of transitions</th>
<th>Number of arcs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(d, k)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(d = 3, k = 4)$</td>
<td>1408</td>
<td>2400</td>
<td>9600</td>
</tr>
<tr>
<td>$(d = 4, k = 3)$</td>
<td>2457</td>
<td>5400</td>
<td>21600</td>
</tr>
<tr>
<td>$(d = 5, k = 3)$</td>
<td>9153</td>
<td>25110</td>
<td>100440</td>
</tr>
</tbody>
</table>

Structural properties

- **ordinary** — all arcs have multiplicity one ❑
- **simple free choice** — all transitions sharing a common input place have no other input place ❑
- **extended free choice** — all transitions sharing a common input place have the same input places ❑
- **state machine** — every transition has exactly one input place and exactly one output place ❑
- **marked graph** — every place has exactly one input transition and exactly one output transition ❑
- **connected** — there is an undirected path between every two nodes (places or transitions) ❑
- **strongly connected** — there is a directed path between every two nodes (places or transitions) ❑
- **source place(s)** — one or more places have no input transitions ❑
- **sink place(s)** — one or more places have no output transitions ❑
- **source transition(s)** — one or more transitions have no input places ❑
- **sink transition(s)** — one or more transitions have no output places ❑
- **loop-free** — no transition has an input place that is also an output place ❑
- **conservative** — for each transition, the number of input arcs equals the number of output arcs ❑
- **subconservative** — for each transition, the number of input arcs equals or exceeds the number of output arcs ❑

**nested units** — places are structured into hierarchically nested sequential units ❑

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(a) stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
(b) stated by CÆSAR.BDD version 2.6 on all 3 instances ((3, 4), (4, 3), (5, 3)).
(c) stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
(d) stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
(e) stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
(f) stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
(g) stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
(h) stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
(i) stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
(j) stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
(k) stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
(l) stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
(m) stated by CÆSAR.BDD version 2.2 on all 3 instances ((3, 4), (4, 3), (5, 3)).
(n) the definition of Nested-Unit Petri Nets (NUPN) is available from [http://mcc.lip6.fr/nupn.php](http://mcc.lip6.fr/nupn.php)
Behavioral properties

safe — in every reachable marking, there is no more than one token on a place .......................... X (a)
deadlock — there exists a reachable marking from which no transition can be fired .......................... X (p)
reversible — from every reachable marking, there is a transition path going back to the initial marking .................................................. X
quasi-live — for every transition \( t \), there exists a reachable marking in which \( t \) can fire ........................ X
live — for every transition \( t \), from every reachable marking, one can reach a marking in which \( t \) can fire ......................... X

Size of the marking graphs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of reachable markings</th>
<th>Number of transition firings</th>
<th>Max. number of tokens per place</th>
<th>Max. number of tokens per marking</th>
</tr>
</thead>
<tbody>
<tr>
<td>((d = 3, k = 4))</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>2784 (^{(q)})</td>
</tr>
<tr>
<td>((d = 4, k = 3))</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>3780 (^{(r)})</td>
</tr>
<tr>
<td>((d = 5, k = 3))</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>14175 (^{(s)})</td>
</tr>
</tbody>
</table>

Other properties

Model is \( 2 \cdot d \cdot p + b \) bounded — the sum of tokens in DCE internal buffer places. Model is P/T-invariant for any natural \( k \) as proven in [1,2]

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\(^{(a)}\) stated by CÆSAR BDD version 2.2 on all 3 instances \((3, 4), (4, 3), (5, 3)\).

\(^{(p)}\) proven in [1,2]; checked by the Tina \url{http://www.laas.fr/tina} tool version 3.3.0 as well as other behavioral properties for small values of parameters \( d, k \).

\(^{(q)}\) number of initial tokens, because the net is conservative.

\(^{(r)}\) number of initial tokens, because the net is conservative.

\(^{(s)}\) number of initial tokens, because the net is conservative.