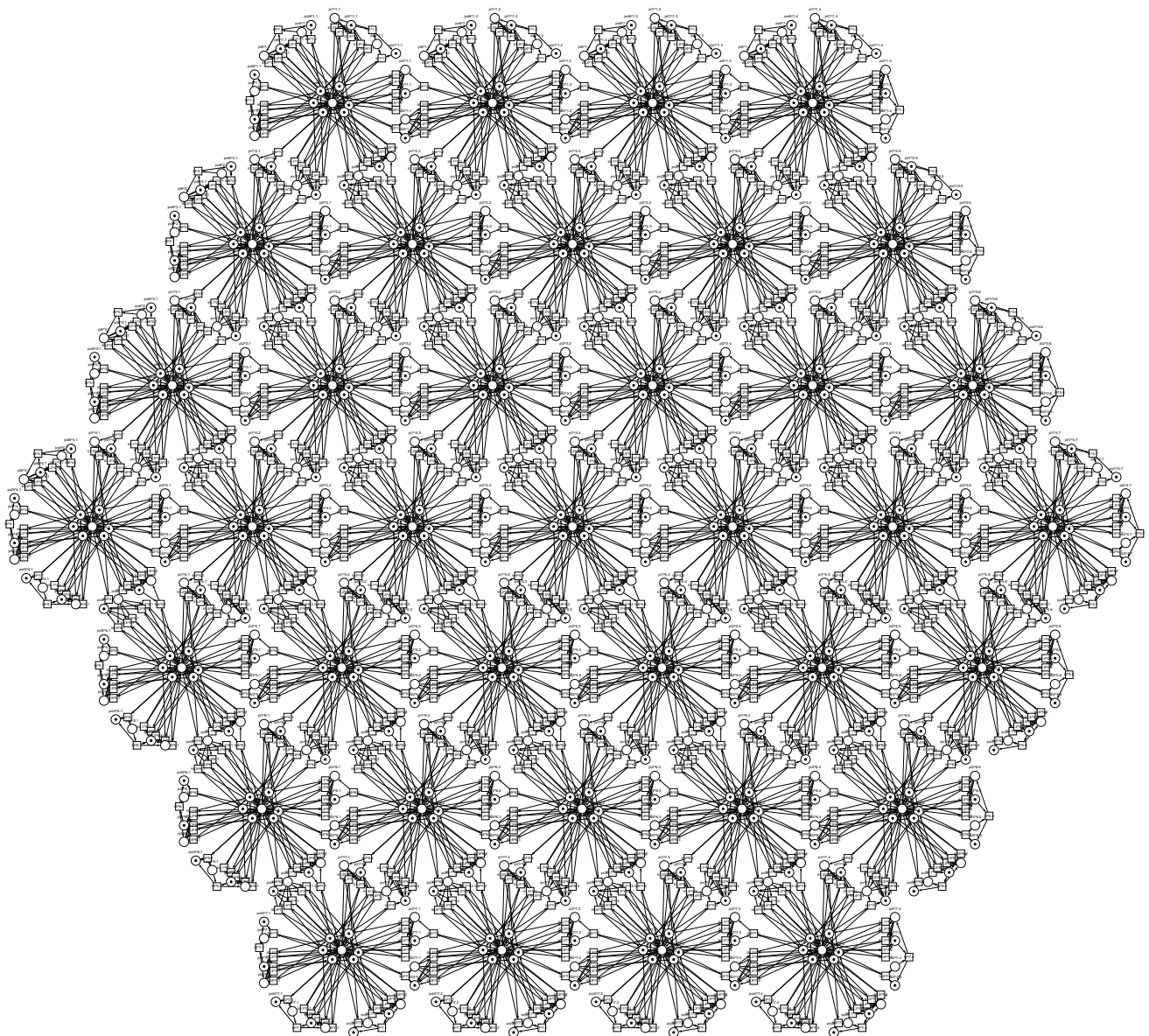


*This form is a summary description of the model entitled “Hexagonal Communication Grid” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.*

## Description

A composition of hexagonal grid is presented. The model is composed of packet switching devices whose ports are situated on sides of a unit-size equilateral hexagon. Each device works in full-duplex mode based on store-and-forward principle. Hexagons are connected via merging contact places situated on common sides. On the edges of hexagons, plug devices are attached. Hexagonal grids are widely applied in mobile networks.



*Graphical representation for  $k = 4$ ,  $p = 1$  and  $b = 0$*

## References

- [1] Zaitsev D.A., Zaitsev I.D., Shmeleva T.R. Infinite Petri Nets as Models of Grids (pp.187-204). Chapter 19 in Mehdi Khosrow-Pour (Ed.) Encyclopedia of Information Science and Technology, Third Edition (10 Volumes). IGI-Global: USA, 2014.
- [2] Shmeleva T.R. Parametric specification of open hexagonal grid. 71 ONAT Conference, December 6-8, 2016, Odessa, p.65-68.

## Scaling parameter

| Parameter name | Parameter description   | Chosen parameter values                                       |
|----------------|---|---|
| $k, p, b$      | $k$ – size of hexagonal grid, $p$ – number of packets in each buffer section, $b$ – available buffer size | (1,1,0), (1,2,6), (2,2,6), (3,1,6), (4,1,0), (5,1,6), (8,1,6) |

## Size of the model

| Parameter   | Number of places   | Number of transitions | Number of arcs      |
|---|--------------------|-----------------------|---------------------|
| $k, p, b$ , with $N_c$ denoting the number of cells in the grid ( $N_c = 3k^2 - 3k + 1$ ) | $19N_c + 24k - 12$ | $36N_c + 12k - 6$     | $144N_c + 48k - 24$ |
| $k = 1, p = 1, b = 0$   | 31                 | 42                    | 168                 |
| $k = 1, p = 2, b = 6$   | 31                 | 42                    | 168                 |
| $k = 2, p = 2, b = 6$   | 169                | 270                   | 1080                |
| $k = 3, p = 1, b = 6$   | 421                | 714                   | 2856                |
| $k = 4, p = 1, b = 0$   | 787                | 1374                  | 5496                |
| $k = 5, p = 1, b = 6$   | 1267               | 2250                  | 9000                |
| $k = 8, p = 1, b = 6$   | 3391               | 6174                  | 24696               |

## Structural properties

- ordinary** — all arcs have multiplicity one ..... ✓
- simple free choice** — all transitions sharing a common input place have no other input place ..... ✗ (a)
- extended free choice** — all transitions sharing a common input place have the same input places ..... ✗ (b)
- state machine** — every transition has exactly one input place and exactly one output place ..... ✗ (c)
- marked graph** — every place has exactly one input transition and exactly one output transition ..... ✗ (d)
- connected** — there is an undirected path between every two nodes (places or transitions) ..... ✓ (e)
- strongly connected** — there is a directed path between every two nodes (places or transitions) ..... ✓ (f)
- source place(s)** — one or more places have no input transitions ..... ✗ (g)
- sink place(s)** — one or more places have no output transitions ..... ✗ (h)
- source transition(s)** — one or more transitions have no input places ..... ✗ (i)
- sink transitions(s)** — one or more transitions have no output places ..... ✗ (j)
- loop-free** — no transition has an input place that is also an output place ..... ✓ (k)
- conservative** — for each transition, the number of input arcs equals the number of output arcs ..... ✓ (l)

- (a) confirmed by [CÆSAR.BDD](#) version 2.7 on all 7 instances (namely: (1,1,0), (1,2,6), (2,2,6), (3,1,6), (4,1,0), (5,1,6), and (8,1,6)).
- (b) confirmed by [CÆSAR.BDD](#) version 2.7 on all 7 instances (namely: (1,1,0), (1,2,6), (2,2,6), (3,1,6), (4,1,0), (5,1,6), and (8,1,6)).
- (c) confirmed by [CÆSAR.BDD](#) version 2.7 on all 7 instances (namely: (1,1,0), (1,2,6), (2,2,6), (3,1,6), (4,1,0), (5,1,6), and (8,1,6)).
- (d) confirmed by [CÆSAR.BDD](#) version 2.7 on all 7 instances (namely: (1,1,0), (1,2,6), (2,2,6), (3,1,6), (4,1,0), (5,1,6), and (8,1,6)).
- (e) confirmed by [CÆSAR.BDD](#) version 2.7 on all 7 instances (namely: (1,1,0), (1,2,6), (2,2,6), (3,1,6), (4,1,0), (5,1,6), and (8,1,6)).
- (f) confirmed by [CÆSAR.BDD](#) version 2.7 on all 7 instances (namely: (1,1,0), (1,2,6), (2,2,6), (3,1,6), (4,1,0), (5,1,6), and (8,1,6)).
- (g) confirmed by [CÆSAR.BDD](#) version 2.7 on all 7 instances (namely: (1,1,0), (1,2,6), (2,2,6), (3,1,6), (4,1,0), (5,1,6), and (8,1,6)).
- (h) confirmed by [CÆSAR.BDD](#) version 2.7 on all 7 instances (namely: (1,1,0), (1,2,6), (2,2,6), (3,1,6), (4,1,0), (5,1,6), and (8,1,6)).
- (i) confirmed by [CÆSAR.BDD](#) version 2.7 on all 7 instances (namely: (1,1,0), (1,2,6), (2,2,6), (3,1,6), (4,1,0), (5,1,6), and (8,1,6)).
- (j) confirmed by [CÆSAR.BDD](#) version 2.7 on all 7 instances (namely: (1,1,0), (1,2,6), (2,2,6), (3,1,6), (4,1,0), (5,1,6), and (8,1,6)).
- (k) confirmed by [CÆSAR.BDD](#) version 2.7 on all 7 instances (namely: (1,1,0), (1,2,6), (2,2,6), (3,1,6), (4,1,0), (5,1,6), and (8,1,6)).
- (l) confirmed by [CÆSAR.BDD](#) version 2.7 on all 7 instances (namely: (1,1,0), (1,2,6), (2,2,6), (3,1,6), (4,1,0), (5,1,6), and (8,1,6)).

**subconservative** — for each transition, the number of input arcs equals or exceeds the number of output arcs ..... ✓<sup>(m)</sup>  
**nested units** — places are structured into hierarchically nested sequential units<sup>(n)</sup> ..... ✗

## Behavioural properties

**safe** — in every reachable marking, there is no more than one token on a place ..... ✗<sup>(o)</sup>  
**deadlock** — there exists a reachable marking from which no transition can be fired ..... ✓  
**reversible** — from every reachable marking, there is a transition path going back to the initial marking ..... ✗  
**quasi-live** — for every transition  $t$ , there exists a reachable marking in which  $t$  can fire ..... ✓<sup>(p)</sup>  
**live** — for every transition  $t$ , from every reachable marking, one can reach a marking in which  $t$  can fire ..... ✗

## Size of the marking graphs

| Parameter             | Number of reachable markings | Number of transition firings | Max. number of tokens per place | Max. number of tokens per marking |
|-----------------------|------------------------------|------------------------------|---------------------------------|-----------------------------------|
| $k = 1, p = 1, b = 0$ | 40193                        | 430884                       | 6                               | 18 <sup>(q)</sup>                 |
| $k = 1, p = 2, b = 6$ | 2664192                      | 39907584                     | 18                              | 30 <sup>(r)</sup>                 |
| $k = 2, p = 2, b = 6$ | ?                            | ?                            | 18                              | 186 <sup>(s)</sup>                |
| $k = 3, p = 1, b = 6$ | ?                            | ?                            | 12                              | 372 <sup>(t)</sup>                |
| $k = 4, p = 1, b = 0$ | ?                            | ?                            | 6                               | 486 <sup>(u)</sup>                |
| $k = 5, p = 1, b = 6$ | ?                            | ?                            | 12                              | 1152 <sup>(v)</sup>               |
| $k = 8, p = 1, b = 6$ | ?                            | ?                            | 12                              | 3132 <sup>(w)</sup>               |

## Other properties

1. To observe a deadlock, the number of packets for blocking a few devices should be provided with parameter  $p$ .
2. Models were analysed using the Tina <http://www.laas.fr/tina> system.

<sup>(m)</sup> confirmed by [CÆSAR.BDD](#) version 2.7 on all 7 instances (namely: (1,1,0), (1,2,6), (2,2,6), (3,1,6), (4,1,0), (5,1,6), and (8,1,6)).

<sup>(n)</sup> the definition of Nested-Unit Petri Nets (NUPN) is available from <http://mcc.lip6.fr/nupn.php>

<sup>(o)</sup> confirmed by [CÆSAR.BDD](#) version 2.7 on all 7 instances (namely: (1,1,0), (1,2,6), (2,2,6), (3,1,6), (4,1,0), (5,1,6), and (8,1,6)).

<sup>(p)</sup> confirmed by [CÆSAR.BDD](#) version 2.7 to be true on 2 instance(s) out of 7, and unknown on the remaining 5 instance(s).

<sup>(q)</sup> number of initial tokens, because the net is conservative.

<sup>(r)</sup> number of initial tokens, because the net is conservative.

<sup>(s)</sup> number of initial tokens, because the net is conservative.

<sup>(t)</sup> number of initial tokens, because the net is conservative.

<sup>(u)</sup> number of initial tokens, because the net is conservative.

<sup>(v)</sup> number of initial tokens, because the net is conservative.

<sup>(w)</sup> number of initial tokens, because the net is conservative.