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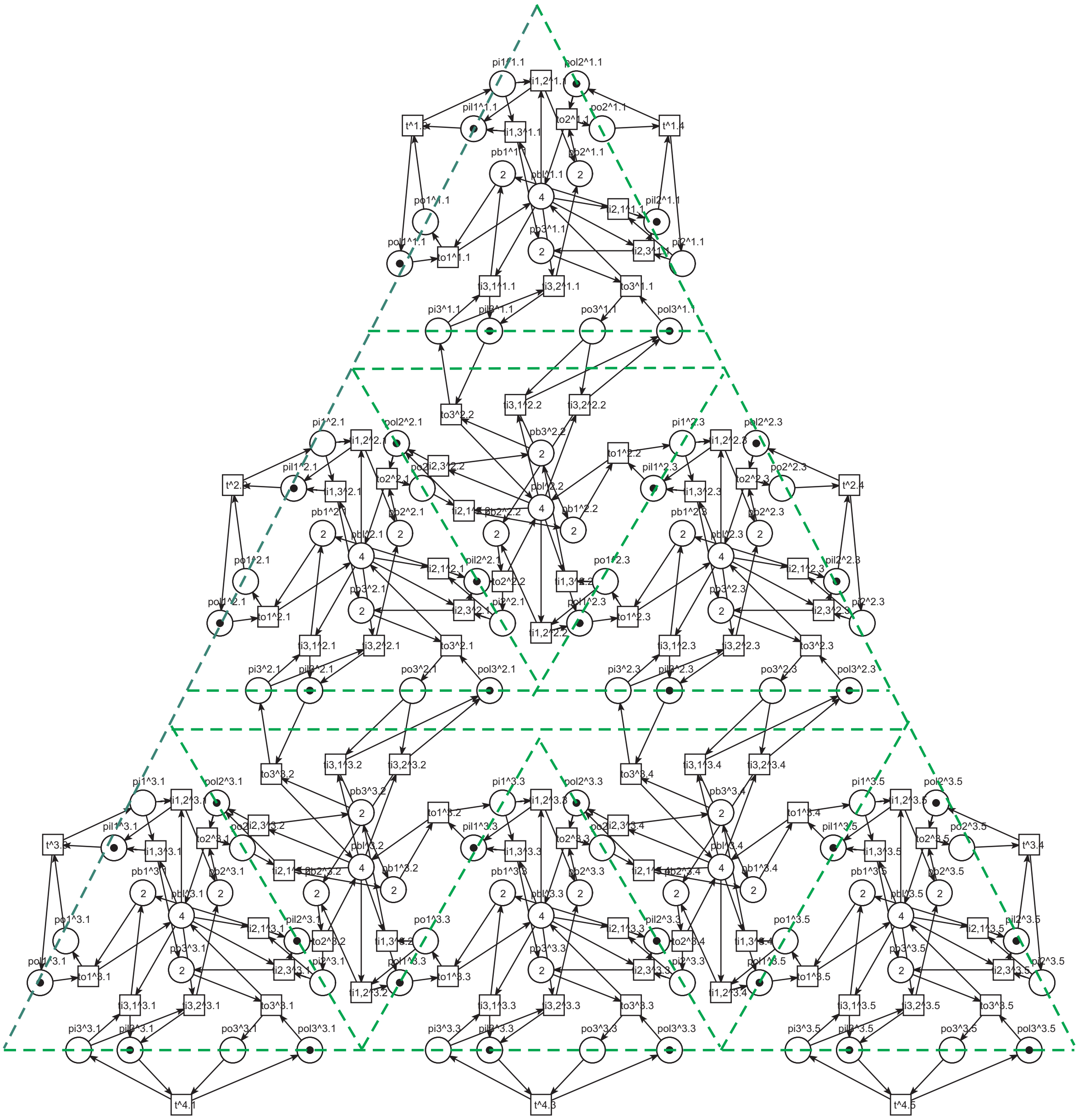
*This form is a summary description of the model entitled "TriangularGrid" proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.*

## Description

The model is composed of packet switching devices whose ports are situated on sides of a unit-size equilateral triangle. Each device works in full-duplex mode based on store-and-forward principle. The grid consists of triangles of two types which interchange in a row. The triangle of the second type is obtained via rotation of the first type triangle by 180 degrees regarding its center. Triangles are connected via merging contact places situated on common sides. On the edges of the triangle, plug devices are attached. Triangular grids are applied in radio- and TV- broadcasting.



*Graphical representation for $k = 3$, $p = 2$ and $b = 4$*

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## References

[1] Zaitsev D.A., Zaitsev I.D., Shmeleva T.R. Infinite Petri Nets as Models of Grids (pp. 187-204). Chapter 19 in Mehdi Khosrow-Pour (Ed.) Encyclopedia of Information Science and Technology, Third Edition (10 Volumes). IGI-Global: USA, 2014.

[2] Shmeleva T.R., Zaitsev D.A., Zaitsev I.D. Analysis of Square Communication Grids via Infinite Petri Nets. Transactions of Odessa National Academy of Telecommunication, no. 1, 2009, p. 27-35.

[3] Shmeleva T.R., Kabachenko I. Analysis of Triangular communication grids via infinite Petri nets. II'09 Conference. Gelendgik (Russia), 2009, p. 178-180.

## Scaling parameter

| Parameter name | Parameter description | Chosen parameter values |
|---|---|---|
| $(k, p, b)$ | k – size of the triangle;<br>p – number of packets in each section of the internal buffer;<br>b – available buffer size. | (1,20,0), (1,50,0), (2,1,1), (3,1,1), (3,2,6), (4,2,2), (5,2,0), (5,4,6) |

## Size of the model

| Parameter | Number of places | Number of transitions | Number of arcs |
|---|---|---|---|
| $(k, p, b)$ | $10k^2 + 6k$ | $9k^2 + 3k$ | $36k^2 + 12k$ |
| $k = 1, p = 20, b = 0$ | 16 | 12 | 48 |
| $k = 1, p = 50, b = 0$ | 16 | 12 | 48 |
| $k = 2, p = 1, b = 1$ | 52 | 42 | 168 |
| $k = 3, p = 1, b = 1$ | 108 | 90 | 360 |
| $k = 3, p = 2, b = 6$ | 108 | 90 | 360 |
| $k = 4, p = 2, b = 2$ | 184 | 156 | 624 |
| $k = 5, p = 2, b = 0$ | 280 | 240 | 960 |
| $k = 5, p = 4, b = 6$ | 280 | 240 | 960 |

## Structural properties

**ordinary** — *all arcs have multiplicity one* ........ ✔
**simple free choice** — *all transitions sharing a common input place have no other input place* ........ ✗ (a)
**extended free choice** — *all transitions sharing a common input place have the same input places* ........ ✗ (b)
**state machine** — *every transition has exactly one input place and exactly one output place* ........ ✗ (c)
**marked graph** — *every place has exactly one input transition and exactly one output transition* ........ ✗ (d)
**connected** — *there is an undirected path between every two nodes (places or transitions)* ........ ✔ (e)
**strongly connected** — *there is a directed path between every two nodes (places or transitions)* ........ ✔ (f)
**source place(s)** — *one or more places have no input transitions* ........ ✗ (g)
**sink place(s)** — *one or more places have no output transitions* ........ ✗ (h)
**source transition(s)** — *one or more transitions have no input places* ........ ✗ (i)
**sink transitions(s)** — *one or more transitions have no output places* ........ ✗ (j)

(a) stated by CÆSAR.BDD version 2.6 on all 8 instances (i.e., the eight triples listed above).
(b) transitions "ti1_2_1_1" and "ti2_1_1_1" share a common input place "pbl_1_1", but only the former transition has input place "pi1_1_1".
(c) stated by CÆSAR.BDD version 2.6 on all 8 instances (i.e., the eight triples listed above).
(d) stated by CÆSAR.BDD version 2.6 on all 8 instances (i.e., the eight triples listed above).
(e) stated by CÆSAR.BDD version 2.6 on all 8 instances (i.e., the eight triples listed above).
(f) stated by CÆSAR.BDD version 2.6 on all 8 instances (i.e., the eight triples listed above).
(g) stated by CÆSAR.BDD version 2.6 on all 8 instances (i.e., the eight triples listed above).
(h) stated by CÆSAR.BDD version 2.6 on all 8 instances (i.e., the eight triples listed above).
(i) stated by CÆSAR.BDD version 2.6 on all 8 instances (i.e., the eight triples listed above).
(j) stated by CÆSAR.BDD version 2.6 on all 8 instances (i.e., the eight triples listed above).

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**loop-free** — *no transition has an input place that is also an output place* ✔ [k]
**conservative** — *for each transition, the number of input arcs equals the number of output arcs* ✔ [l]
**subconservative** — *for each transition, the number of input arcs equals or exceeds the number of output arcs* ✔ [m]
**nested units** — *places are structured into hierarchically nested sequential units* [n] ✗

## Behavioural properties

**safe** — *in every reachable marking, there is no more than one token on a place* ✗ [o]
**deadlock** — *there exists a reachable marking from which no transition can be fired* ✔
**reversible** — *from every reachable marking, there is a transition path going back to the initial marking* ✗
**quasi-live** — *for every transition t, there exists a reachable marking in which t can fire* ✔ [p]
**live** — *for every transition t, from every reachable marking, one can reach a marking in which t can fire* ✗

## Size of the marking graphs

| Parameter | Number of reachable markings | Number of transition firings | Max. number of tokens per place | Max. number of tokens per marking |
|---|---|---|---|---|
| $k = 1, p = 20, b = 0$ | 109552 | 566712 | 60 | 66 [q] |
| $k = 1, p = 50, b = 0$ | 705712 | 3683592 | 150 | 156 [r] |
| $k = 2, p = 1, b = 1$ | $\geq$ 1.09633e+09 [s] | ? | 4 | 34 [t] |
| $k = 3, p = 1, b = 1$ | $\geq$ 3.81926e+16 [u] | ? | 4 | 72 [v] |
| $k = 3, p = 2, b = 6$ | ? | ? | 12 | 144 [w] |
| $k = 4, p = 2, b = 2$ | ? | ? | 8 | 188 [x] |
| $k = 5, p = 2, b = 0$ | ? | ? | 6 | 240 [y] |
| $k = 5, p = 4, b = 6$ | ? | ? | 18 | 540 [z] |

## Other properties

1. To observe a deadlock, the number of packets for blocking a few devices should be provided with parameter $p$.
2. Models were analysed using the Tina http://www.laas.fr/tina system.

[k] stated by CÆSAR.BDD version 2.6 on all 8 instances (i.e., the eight triples listed above).
[l] stated by CÆSAR.BDD version 2.6 on all 8 instances (i.e., the eight triples listed above).
[m] stated by CÆSAR.BDD version 2.6 on all 8 instances (i.e., the eight triples listed above).
[n] the definition of Nested-Unit Petri Nets (NUPN) is available from `http://mcc.lip6.fr/nupn.php`
[o] stated by CÆSAR.BDD version 2.6 on all 8 instances (i.e., the eight triples listed above).
[p] stated by CÆSAR.BDD version 2.6 to be true on 5 instance(s) out of 8, and unknown on the remaining 3 instance(s).
[q] number of initial tokens, because the net is conservative.
[r] number of initial tokens, because the net is conservative.
[s] stated by CÆSAR.BDD version 2.6.
[t] number of initial tokens, because the net is conservative.
[u] stated by CÆSAR.BDD version 2.6.
[v] number of initial tokens, because the net is conservative.
[w] number of initial tokens, because the net is conservative.
[x] number of initial tokens, because the net is conservative.
[y] number of initial tokens, because the net is conservative.
[z] number of initial tokens, because the net is conservative.