This form is a summary description of the model entitled “Solitaire” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.

Description

Solitaire is a popular board game requiring non-obvious solution strategies; see [wiki] for the rules of the game. The objective of the Petri nets is to generate one/some/all strategies (paths) to reach a solution, i.e., a state where just one stone is left. The auxiliary place \textit{counter} gives the current number of stones on the board; added to simplify the specification of the target state (any state with \textit{counter} = 1). Solitaire is played on different boards; we give Petri nets for the most popular ones: square board (0), English board (1), French board (2), each in two versions: with/out counter [H05]. The existence of a solution may depend on the initially empty field; all initial markings have been chosen to enable a solution. Encoding this game as coloured Petri net would permit the generation of arbitrary boards of scalable size.

References


Scaling parameter

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Parameter description</th>
<th>Chosen parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>shape and size of the board</td>
<td>5 × 5 square board (0), 7 × 7 English board (1), 7 × 7 French board (3)</td>
</tr>
</tbody>
</table>
Size of the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of places</th>
<th>Number of transitions</th>
<th>Number of arcs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B = 0$</td>
<td>50</td>
<td>84</td>
<td>456</td>
</tr>
<tr>
<td>$B = 0$, with counter</td>
<td>51</td>
<td>84</td>
<td>540</td>
</tr>
<tr>
<td>$B = 1$</td>
<td>66</td>
<td>76</td>
<td>456</td>
</tr>
<tr>
<td>$B = 1$, with counter</td>
<td>67</td>
<td>76</td>
<td>532</td>
</tr>
<tr>
<td>$B = 2$</td>
<td>74</td>
<td>92</td>
<td>552</td>
</tr>
<tr>
<td>$B = 2$, with counter</td>
<td>75</td>
<td>92</td>
<td>644</td>
</tr>
</tbody>
</table>

Structural properties

- ordinary — all arcs have multiplicity one
- simple free choice — all transitions sharing a common input place have no other input place
- extended free choice — all transitions sharing a common input place have the same input places
- state machine — every transition has exactly one input place and exactly one output place
- marked graph — every place has exactly one input transition and exactly one output transition
- connected — there is an undirected path between every two nodes (places or transitions)
- strongly connected — there is a directed path between every two nodes (places or transitions)
- source place(s) — one or more places have no input transitions
- sink place(s) — one or more places have no output transitions
- source transition(s) — one or more transitions have no input places
- sink transitions(s) — one or more transitions have no output places
- loop-free — no transition has an input place that is also an output place
- conservative — for each transition, the number of input arcs equals the number of output arcs
- subconservative — for each transition, the number input arcs equals or exceeds the number of output arcs
- nested units — places are structured into hierarchically nested sequential units

Behavioural properties

- safe — in every reachable marking, there is no more than one token on a place
- deadlock — there exists a reachable marking from which no transition can be fired
- reversible — from every reachable marking, there is a transition path going back to the initial marking
- quasi-live — for every transition $t$, there exists a reachable marking in which $t$ can fire
- live — for every transition $t$, from every reachable marking, one can reach a marking in which $t$ can fire

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(a) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(b) stated by CÆSAR.BDD version 2.6 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(c) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(d) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(e) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(f) stated by CÆSAR.BDD version 2.0 to be false on all 3 instances with counters, and true on all 3 instances without counters.
(g) stated by CÆSAR.BDD version 2.0 to be true on all 3 instances with counters, and false on all 3 instances without counters.
(h) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(i) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(j) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(k) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(l) stated by CÆSAR.BDD version 2.0 to be false on all 3 instances with counters, and true on all 3 instances without counters.
(m) stated by CÆSAR.BDD version 2.0 to be false on all 3 instances with counters, and unknown on the remaining 3 instance(s).

(a) the nets corresponding to instances without counters are safe because they are covered with P-invariants having a single token in the initial place – found by CÆSAR.BDD version 2.0 to be false on all 3 instances with counters, and unknown on the remaining 3 instance(s).

(p) special deadlocks (dead states) correspond to the solutions we are looking for; confirmed at MCC’2014 by Lola and Tapaal on all 6 instances.
Size of the marking graphs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of reachable markings</th>
<th>Number of transition firings</th>
<th>Max. number of tokens per place</th>
<th>Max. number of tokens per marking</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B = 0$</td>
<td>$1.6098 \times 10^7$ (q)</td>
<td>$2.1396 \times 10^5$ (r)</td>
<td>1 (s)</td>
<td>25 (t)</td>
</tr>
<tr>
<td>$B = 0$, with counter</td>
<td>?</td>
<td>?</td>
<td>24</td>
<td>49 (u)</td>
</tr>
<tr>
<td>$B = 1$</td>
<td>?</td>
<td>?</td>
<td>1</td>
<td>33 (v)</td>
</tr>
<tr>
<td>$B = 1$, with counter</td>
<td>?</td>
<td>?</td>
<td>32</td>
<td>65 (w)</td>
</tr>
<tr>
<td>$B = 2$</td>
<td>?</td>
<td>?</td>
<td>1</td>
<td>37 (x)</td>
</tr>
<tr>
<td>$B = 2$, with counter</td>
<td>?</td>
<td>?</td>
<td>36</td>
<td>73 (y)</td>
</tr>
</tbody>
</table>

Other properties

Deadlocks (dead states) which correspond to a solution can be identified by: sum over all places $T_{i,j} = 1$, or counter=0. All places are covered by 1-P-invariants, except the counter place. All nets enjoy some symmetries.

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(q) computed at MCC’2014 by Marcie, PNMC, and PNXDD; exact value: 16,098,428.
(r) computed at MCC’2014 by Marcie; exact value: 213,958,152.
(s) computed at MCC’2014 by Marcie and PNMC.
(t) number of initial tokens, because the net is sub-conservative.
(u) number of initial tokens, because the net is sub-conservative.
(v) number of initial tokens, because the net is sub-conservative.
(w) number of initial tokens, because the net is sub-conservative.
(x) number of initial tokens, because the net is sub-conservative.
(y) number of initial tokens, because the net is sub-conservative.