Model: Solitaire
Type: P/T Net
Origin: Academic

since
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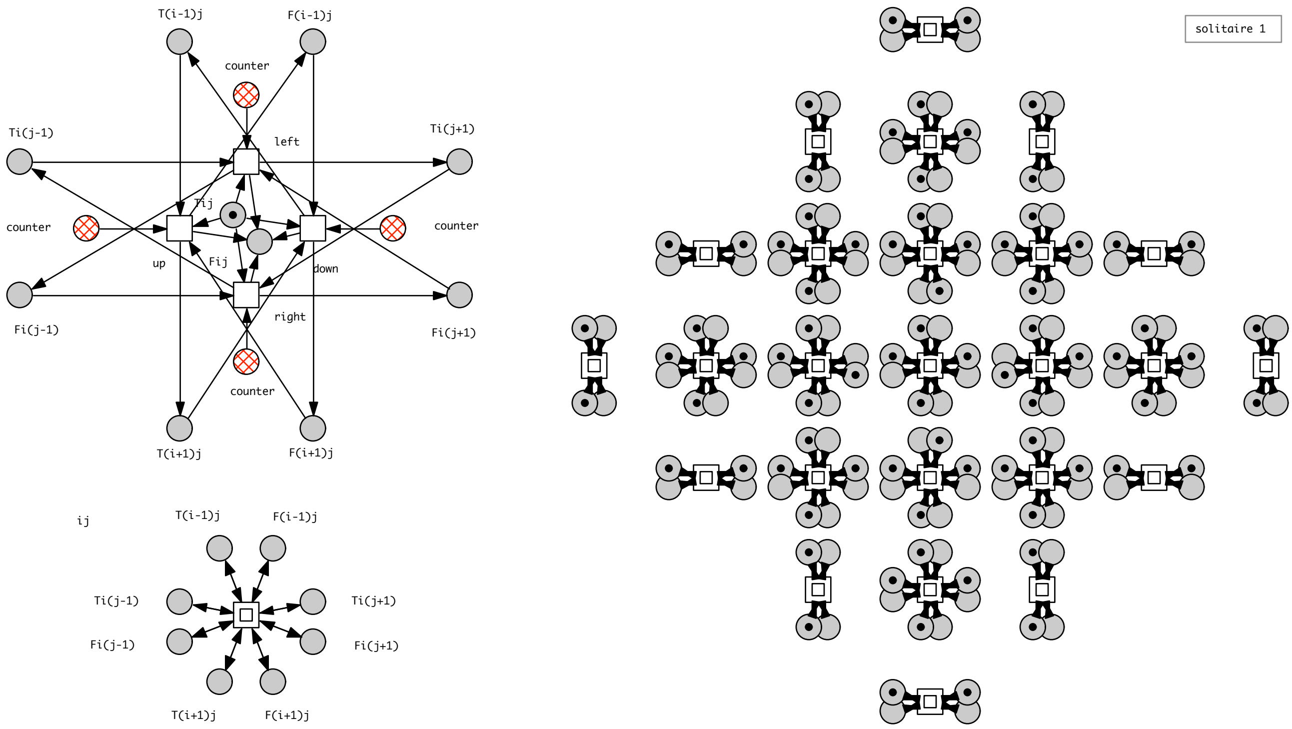
This form is a summary description of the model entitled "Solitaire" proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.

Description

Solitaire is a popular board game requiring non-obvious solution strategies; see [wiki] for the rules of the game. The objective of the Petri nets is to generate one/some/all strategies (paths) to reach a solution, i.e., a state where just one stone is left. The auxiliary place *counter* gives the current number of stones on the board; added to simplify the specification of the target state (any state with *counter* = 1). Solitaire is played on different boards; we give Petri nets for the most popular ones: square board (0), English board (1), French board (2), each in two versions: with/out counter [H05]. The existence of a solution may depend on the initially empty field; all initial markings have been chosen to enable a solution. Encoding this game as coloured Petri net would permit the generation of arbitrary boards of scalable size.



General solitaire pattern for one field (left), and its composition to the 7×7 English board (right).

References

H05 M Heiner: About some Applications of Petri Net Theory - My Petri Net Picture Book; Talk, Adventmatik 2003, Paderborn, December 2003, http://www-dssz.informatik.tu-cottbus.de/publications/slides/2003_paderborn_pn_applications.sld.pdf.

Wiki Wikipedia: Peg solitaire; http://en.wikipedia.org/wiki/Peg_solitaire, last access 12/2013.

Scaling parameter

Parameter name	Parameter description	Chosen parameter values
B	shape and size of the board	5×5 square board (0), 7×7 English board (1), 7×7 French board (3)

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Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
$B = 0$	50	84	456
$B = 0$, with counter	51	84	540
$B = 1$	66	76	456
$B = 1$, with counter	67	76	532
$B = 2$	74	92	552
$B = 2$, with counter	75	92	644

Structural properties

ordinary — *all arcs have multiplicity one* .... ✔
simple free choice — *all transitions sharing a common input place have no other input place* .... ✗ (a)
extended free choice — *all transitions sharing a common input place have the same input places* .... ✗ (b)
state machine — *every transition has exactly one input place and exactly one output place* .... ✗ (c)
marked graph — *every place has exactly one input transition and exactly one output transition* .... ✗ (d)
connected — *there is an undirected path between every two nodes (places or transitions)* .... ✔ (e)
strongly connected — *there is a directed path between every two nodes (places or transitions)* .... ? (f)
source place(s) — *one or more places have no input transitions* .... ? (g)
sink place(s) — *one or more places have no output transitions* .... ✗ (h)
source transition(s) — *one or more transitions have no input places* .... ✗ (i)
sink transitions(s) — *one or more transitions have no output places* .... ✗ (j)
loop-free — *no transition has an input place that is also an output place* .... ✔ (k)
conservative — *for each transition, the number of input arcs equals the number of output arcs* .... ? (l)
subconservative — *for each transition, the number of input arcs equals or exceeds the number of output arcs* .... ✔ (m)
nested units — *places are structured into hierarchically nested sequential units* (n) .... ✗

Behavioural properties

safe — *in every reachable marking, there is no more than one token on a place* .... ? (o)
deadlock — *there exists a reachable marking from which no transition can be fired* .... ✔ (p)
reversible — *from every reachable marking, there is a transition path going back to the initial marking* .... ✗
quasi-live — *for every transition t, there exists a reachable marking in which t can fire* .... ✔
live — *for every transition t, from every reachable marking, one can reach a marking in which t can fire* .... ✗

(a) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(b) stated by CÆSAR.BDD version 2.6 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(c) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(d) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(e) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(f) stated by CÆSAR.BDD version 2.0 to be false on all 3 instances with counters, and true on all 3 instances without counters.
(g) stated by CÆSAR.BDD version 2.0 to be true on all 3 instances with counters, and false on all 3 instances without counters.
(h) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(i) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(j) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(k) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(l) stated by CÆSAR.BDD version 2.0 to be false on all 3 instances with counters, and true on all 3 instances without counters.
(m) stated by CÆSAR.BDD version 2.0 on all 6 instances ($B \in \{0, 1, 2\}$, with and without counter).
(n) the definition of Nested-Unit Petri Nets (NUPN) is available from `http://mcc.lip6.fr/nupn.php`
(o) the nets corresponding to instances without counters are safe because they are covered with P-invariants having a single token in the initial place – found by CÆSAR.BDD version 2.0 to be false on all 3 instances with counters, and unknown on the remaining 3 instance(s).
(p) special deadlocks (dead states) correspond to the solutions we are looking for; confirmed at MCC'2014 by Lola and Tapaal on all 6 instances.

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Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
$B = 0$	1.6098×10^7 [q]	2.1396×10^8 [r]	1 [s]	25 [t]
$B = 0$, with counter	?	?	24	49 [u]
$B = 1$	?	?	1	33 [v]
$B = 1$, with counter	?	?	32	65 [w]
$B = 2$	?	?	1	37 [x]
$B = 2$, with counter	?	?	36	73 [y]

Other properties

Deadlocks (dead states) which correspond to a solution can be identified by: sum over all places $T_{i,j} = 1$, or counter=0. All places are covered by 1-P-invariants, except the counter place. All nets enjoy some symmetries.

(q) computed at MCC'2014 by Marcie, PNMC, and PNXDD; exact value: 16,098,428.
(r) computed at MCC'2014 by Marcie; exact value: 213,958,152.
(s) computed at MCC'2014 by Marcie and PNMC.
(t) number of initial tokens, because the net is sub-conservative.
(u) number of initial tokens, because the net is sub-conservative.
(v) number of initial tokens, because the net is sub-conservative.
(w) number of initial tokens, because the net is sub-conservative.
(x) number of initial tokens, because the net is sub-conservative.
(y) number of initial tokens, because the net is sub-conservative.