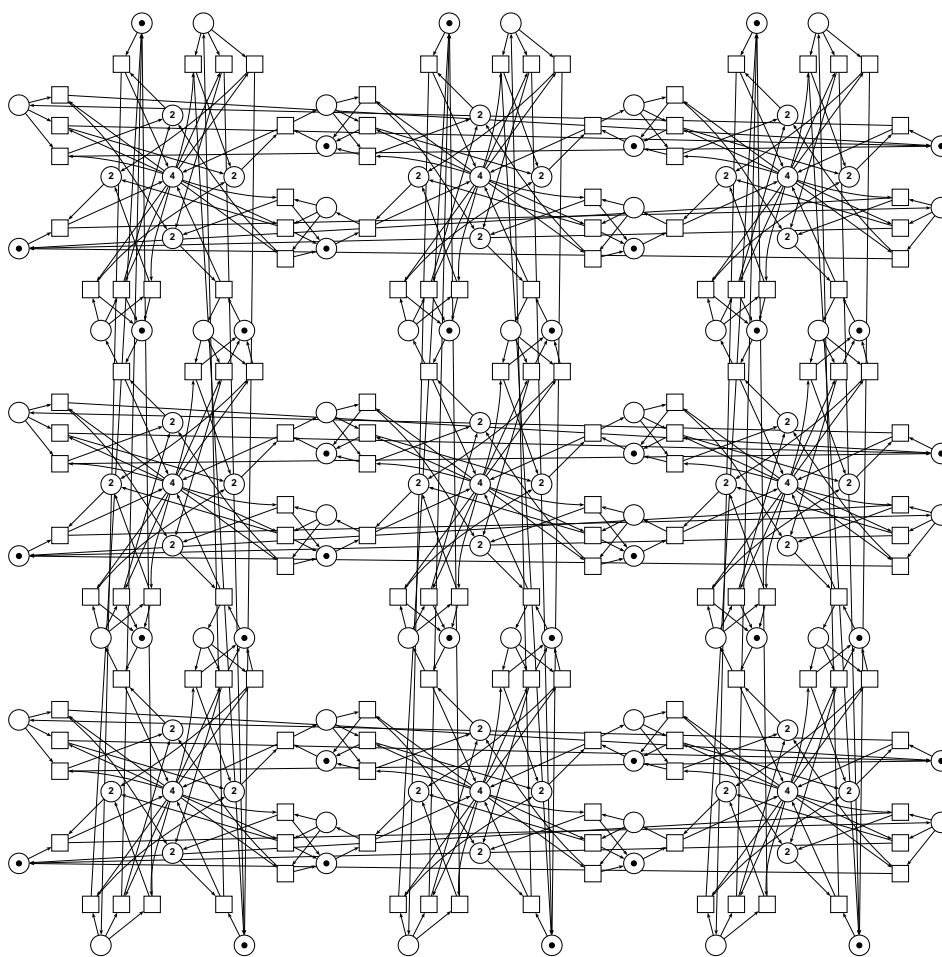


*This form is a summary description of the model entitled “HypertorusGrid” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.*

## Description

A hypertorus is obtained from a hypercube via closing (connecting) opposite facets in each dimension. A cell of hypertorus grid represents a computing and packet switching device with ports situated on facets of the unit-sized hypercube. A device works in full-duplex mode using store-and-forward principle with limited capacity of buffer. Neighboring cells are connected via merging contact places situated on common facets.



*Graphical representation for  $d = 2$ ,  $k = 3$ ,  $p = 2$ , and  $b = 4$ .*

## References

- [1] Zaitsev D.A., Zaitsev I.D., Shmeleva T.R. [Infinite Petri Nets as Models of Grids](#). Chapter 19 in Mehdi Khosrow-Pour (Ed.) Encyclopedia of Information Science and Technology, Third Edition (10 Volumes). IGI-Global: USA, pp. 187–204, 2014.
- [2] Zaitsev D.A. [Verification of Computing Grids with Special Edge Conditions by Infinite Petri Nets](#), Automatic Control and Computer Sciences, 2013, vol. 47, no. 7, pp.403–412.
- [3] Zaitsev D.A. Generator of hypertorus Petri net models. <http://github.com/dazeorgacm/htgen>

## Scaling parameter

Parameter name	Parameter description	Chosen parameter values
$(d, k, p, b)$	$d$ – dimension of hypertorus; $k$ – size in each dimension, totally $k^d$ nodes; $p$ – number of packets in each section of the internal buffer; $b$ – available buffer size. $d$ and $k$ influence the Petri net structure while $p$ and $b$ define its initial marking.	$(2,1,8,0)$ , $(2,2,1,0)$ , $(2,3,2,4)$ , $(3,3,2,6)$ , $(4,3,2,8)$ , $(5,3,2,10)$

## Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
$(d, k, p, b)$	$6 \cdot d \cdot k^d + k^d$	$4 \cdot d^2 \cdot k^d$	$16 \cdot d^2 \cdot k^d$
$(2, 1, 8, 0)$	13	16	64
$(2, 2, 1, 0)$	52	64	256
$(2, 3, 2, 4)$	117	144	576
$(3, 3, 2, 6)$	513	972	3888
$(4, 3, 2, 8)$	2025	5184	20736
$(5, 3, 2, 10)$	7533	24300	97200

## Structural properties

<b>ordinary</b> — all arcs have multiplicity one .....	✓
<b>simple free choice</b> — all transitions sharing a common input place have no other input place .....	✗ (a)
<b>extended free choice</b> — all transitions sharing a common input place have the same input places .....	✗ (b)
<b>state machine</b> — every transition has exactly one input place and exactly one output place .....	✗ (c)
<b>marked graph</b> — every place has exactly one input transition and exactly one output transition .....	✗ (d)
<b>connected</b> — there is an undirected path between every two nodes (places or transitions) .....	✓ (e)
<b>strongly connected</b> — there is a directed path between every two nodes (places or transitions) .....	✓ (f)
<b>source place(s)</b> — one or more places have no input transitions .....	✗ (g)
<b>sink place(s)</b> — one or more places have no output transitions .....	✗ (h)
<b>source transition(s)</b> — one or more transitions have no input places .....	✗ (i)
<b>sink transitions(s)</b> — one or more transitions have no output places .....	✗ (j)
<b>loop-free</b> — no transition has an input place that is also an output place .....	✓ (k)
<b>conservative</b> — for each transition, the number of input arcs equals the number of output arcs .....	✓ (l)
<b>subconservative</b> — for each transition, the number of input arcs equals or exceeds the number of output arcs .....	✓ (m)
<b>nested units</b> — places are structured into hierarchically nested sequential units <sup>(n)</sup> .....	✗

(a) stated by CÆSAR.BDD version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(b) stated by CÆSAR.BDD version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(c) stated by CÆSAR.BDD version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(d) stated by CÆSAR.BDD version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(e) stated by CÆSAR.BDD version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(f) stated by CÆSAR.BDD version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(g) stated by CÆSAR.BDD version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(h) stated by CÆSAR.BDD version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(i) stated by CÆSAR.BDD version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(j) stated by CÆSAR.BDD version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(k) stated by CÆSAR.BDD version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(l) stated by CÆSAR.BDD version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(m) stated by CÆSAR.BDD version 2.6 on all 6 instances (i.e., the six quadruples listed above).

(n) the definition of Nested-Unit Petri Nets (NUPN) is available from <http://mcc.lip6.fr/nupn.php>

## Behavioural properties

- safe** — *in every reachable marking, there is no more than one token on a place* ..... X <sup>(o)</sup>  
**deadlock** — *there exists a reachable marking from which no transition can be fired* ..... ✓  
**reversible** — *from every reachable marking, there is a transition path going back to the initial marking* ..... X  
**quasi-live** — *for every transition  $t$ , there exists a reachable marking in which  $t$  can fire* ..... ✓ <sup>(p)</sup>  
**live** — *for every transition  $t$ , from every reachable marking, one can reach a marking in which  $t$  can fire* ..... X

## Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
(2, 1, 8, 0)	87552	667632	32	36
(2, 2, 1, 0)	$\geq 7.50898e+08$ <sup>(q)</sup>	?	?	$\geq 32$ <sup>(r)</sup>
(2, 3, 2, 4)	?	?	?	144 <sup>(s)</sup>
(3, 3, 2, 6)	?	?	?	648 <sup>(t)</sup>
(4, 3, 2, 8)	?	?	?	2592 <sup>(u)</sup>
(5, 3, 2, 10)	?	?	?	9720 <sup>(v)</sup>

## Other properties

To observe a deadlock, there should be enough packets ( $p$ ) compared to available buffer size ( $b$ ) to block a couple or more devices [1,2].

Using *htgen* [3], a model in format .net of Tina modeling system (<http://www.laas.fr/tina>) was built for given values of parameters.

<sup>(o)</sup> stated by CÆSAR.BDD version 2.6 on all 6 instances (i.e., the six quadruples listed above).

<sup>(p)</sup> stated by CÆSAR.BDD version 2.6 to be true on 2 instance(s) out of 6, and unknown on the remaining 4 instance(s).

<sup>(q)</sup> stated by CÆSAR.BDD version 2.6.

<sup>(r)</sup> lower bound given by the number of initial tokens.

<sup>(s)</sup> number of initial tokens, because the net is conservative.

<sup>(t)</sup> number of initial tokens, because the net is conservative.

<sup>(u)</sup> number of initial tokens, because the net is conservative.

<sup>(v)</sup> number of initial tokens, because the net is conservative.