This form is a summary description of the model entitled “HypercubeCommunicationGrid” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.

Description

Hypercube communication grid model [1,2] is composed of nodes which represent data communication equipment (DCE) implementing packet forwarding based on store-and-forward principle. Each DCE has ports, situated on facets of a unit size hypercube, which work in full-duplex mode. Data terminal equipment (DTE) is attached on the hypercube borders. Each DTE receives and sends packets.

Remind that, a \( d \)-dimension hypercube has \( 2 \cdot d \) facets each represents a \( (d - 1) \)-dimension hypercube.

DCE index \((i_1, i_2, ..., i_d)\), where \(1 \leq i_j \leq k\), \(1 \leq j \leq d\), reflects its location within hypercube. Port index \((r, n)\) consists of dimension number \(1 \leq r \leq d\), a facet is perpendicular to, and direction number \(1 \leq n \leq 2\), where \(n = 1\) represents the direction to the origin of coordinates and \(n = 2\) represents the direction to infinity.

DCE model contains an internal buffer represented with \(2 \cdot d + 1\) places: the available buffer size and buffer sections for storing packets forwarded to the corresponding ports.

Each of \(2 \cdot d\) DCE ports has two tracts: input and output. Memory of a tract is represented with two places – the tract buffer and the tract buffer available capacity (usually equal to unit). An output tract work is modeled by a single transition taking a packet from the corresponding section of the internal buffer and putting it into the tract buffer. An input tract work is modeled by \(2 \cdot d - 1\) transitions forwarding arrived packet from the tract buffer to the corresponding section of the internal buffer except of the arrival port number.

A hypercube is composed via merging tract places of neighbor DCE which has a common facet: input tract of one DCE with output tract of the other DCE and vice versa.

On the borders, which constitute \(2 \cdot d\) hypercubes of dimension \(d - 1\), DTE models are attached. A simple DTE model is represented with a single transition that receives a packet from a neighbor DCE output tract and sends a packet into the neighbor DCE input tract.

For planar case when \(d = 2\), models are described in [1,3] with simplified notation of ports.

References

[4] A C program that generates \(k^d\) hypercube can be downloaded from [http://daze.ho.ua/tinaz.zip](http://daze.ho.ua/tinaz.zip)
## Scaling parameter

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Parameter description</th>
<th>Chosen parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d,k,p,b$</td>
<td>$d$ is the number of dimensions; $k$ is the hypercube size of $k^d$ DCE nodes and $2 \cdot d \cdot k^{d-1}$ DTE nodes; $p$ is the number of packets in each section of internal buffer; $b$ is the available size of internal buffer; $p$ and $b$ define the initial marking and do not affect the model structure.</td>
<td>$(d,k) = (3,4),(4,3),(5,3)$ with $p = k$ and $b = d \cdot k$</td>
</tr>
</tbody>
</table>

## Size of the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of places</th>
<th>Number of transitions</th>
<th>Number of arcs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(d,k)$</td>
<td>$P = 6 \cdot d \cdot k^d + k^d + 4 \cdot d \cdot k^{d-1}$</td>
<td>$T = 4 \cdot d^2 \cdot k^d + 2 \cdot d \cdot k^{d-1}$</td>
<td>$A = 16 \cdot d^2 \cdot k^d + 8 \cdot d \cdot k^{d-1}$</td>
</tr>
<tr>
<td>$(d = 3,k = 4)$</td>
<td>1408</td>
<td>2400</td>
<td>9600</td>
</tr>
<tr>
<td>$(d = 4,k = 3)$</td>
<td>2457</td>
<td>5400</td>
<td>21600</td>
</tr>
<tr>
<td>$(d = 5,k = 3)$</td>
<td>9153</td>
<td>25110</td>
<td>100440</td>
</tr>
</tbody>
</table>

## Structural properties

- ordinary — all arcs have multiplicity one
- simple free choice — all transitions sharing a common input place have no other input place
- extended free choice — all transitions sharing a common input place have the same input places
- state machine — every transition has exactly one input place and exactly one output place
- marked graph — every place has exactly one input transition and exactly one output transition
- connected — there is an undirected path between every two nodes (places or transitions)
- strongly connected — there is a directed path between every two nodes (places or transitions)
- source place(s) — one or more places have no input transitions
- sink place(s) — one or more places have no output transitions
- source transition(s) — one or more transitions have no input places
- sink transition(s) — one or more transitions have no output places
- loop-free — no transition has an input place that is also an output place
- conservative — for each transition, the number of input arcs equals the number of output arcs
- subconservative — for each transition, the number of input arcs equals or exceeds the number of output arcs
- nested units — places are structured into hierarchically nested sequential units?

(a) stated by CÆSAR.BDD version 2.2 on all 3 instances $(3,4),(4,3),(5,3)$.
(b) stated by CÆSAR.BDD version 2.6 on all 3 instances $(3,4),(4,3),(5,3)$.
(c) stated by CÆSAR.BDD version 2.2 on all 3 instances $(3,4),(4,3),(5,3)$.
(d) stated by CÆSAR.BDD version 2.2 on all 3 instances $(3,4),(4,3),(5,3)$.
(e) stated by CÆSAR.BDD version 2.2 on all 3 instances $(3,4),(4,3),(5,3)$.
(f) stated by CÆSAR.BDD version 2.2 on all 3 instances $(3,4),(4,3),(5,3)$.
(g) stated by CÆSAR.BDD version 2.2 on all 3 instances $(3,4),(4,3),(5,3)$.
(h) stated by CÆSAR.BDD version 2.2 on all 3 instances $(3,4),(4,3),(5,3)$.
(i) stated by CÆSAR.BDD version 2.2 on all 3 instances $(3,4),(4,3),(5,3)$.
(j) stated by CÆSAR.BDD version 2.2 on all 3 instances $(3,4),(4,3),(5,3)$.
(k) stated by CÆSAR.BDD version 2.2 on all 3 instances $(3,4),(4,3),(5,3)$.
(l) stated by CÆSAR.BDD version 2.2 on all 3 instances $(3,4),(4,3),(5,3)$.
(m) stated by CÆSAR.BDD version 2.2 on all 3 instances $(3,4),(4,3),(5,3)$.
(n) the definition of Nested-Unit Petri Nets (NUPN) is available from [http://mcc.lip6.fr/nupn.php](http://mcc.lip6.fr/nupn.php)
Behavioural properties

safe — in every reachable marking, there is no more than one token on a place ......................................... \(\times\) (o)
deadlock — there exists a reachable marking from which no transition can be fired ........................................ \(\times\) (p)
reversible — from every reachable marking, there is a transition path going back to the initial marking ................ \(\times\)
quasi-live — for every transition \(t\), there exists a reachable marking in which \(t\) can fire ........................................ \(\checkmark\)
live — for every transition \(t\), from every reachable marking, one can reach a marking in which \(t\) can fire ...................... \(\times\)

Size of the marking graphs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of reachable markings</th>
<th>Number of transition firings</th>
<th>Max. number of tokens per place</th>
<th>Max. number of tokens per marking</th>
</tr>
</thead>
<tbody>
<tr>
<td>((d = 3, k = 4))</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>2784 (^{(q)})</td>
</tr>
<tr>
<td>((d = 4, k = 3))</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>3780 (^{(r)})</td>
</tr>
<tr>
<td>((d = 5, k = 3))</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>14175 (^{(s)})</td>
</tr>
</tbody>
</table>

Other properties

Model is \(2 \cdot d \cdot p + b\) bounded — the sum of tokens in DCE internal buffer places. Model is P/T-invariant for any natural \(k\) as proven in \([1,2]\)

\(^{(o)}\) stated by \textsc{caesar.BDD} version 2.2 on all 3 instances \((3, 4), (4, 3), (5, 3)\).

\(^{(p)}\) proven in \([1,2]\); checked by the Tina \url{http://www.laas.fr/tina} tool version 3.3.0 as well as other behavioural properties for small values of parameters \(d, k\).

\(^{(q)}\) number of initial tokens, because the net is conservative.

\(^{(r)}\) number of initial tokens, because the net is conservative.

\(^{(s)}\) number of initial tokens, because the net is conservative.