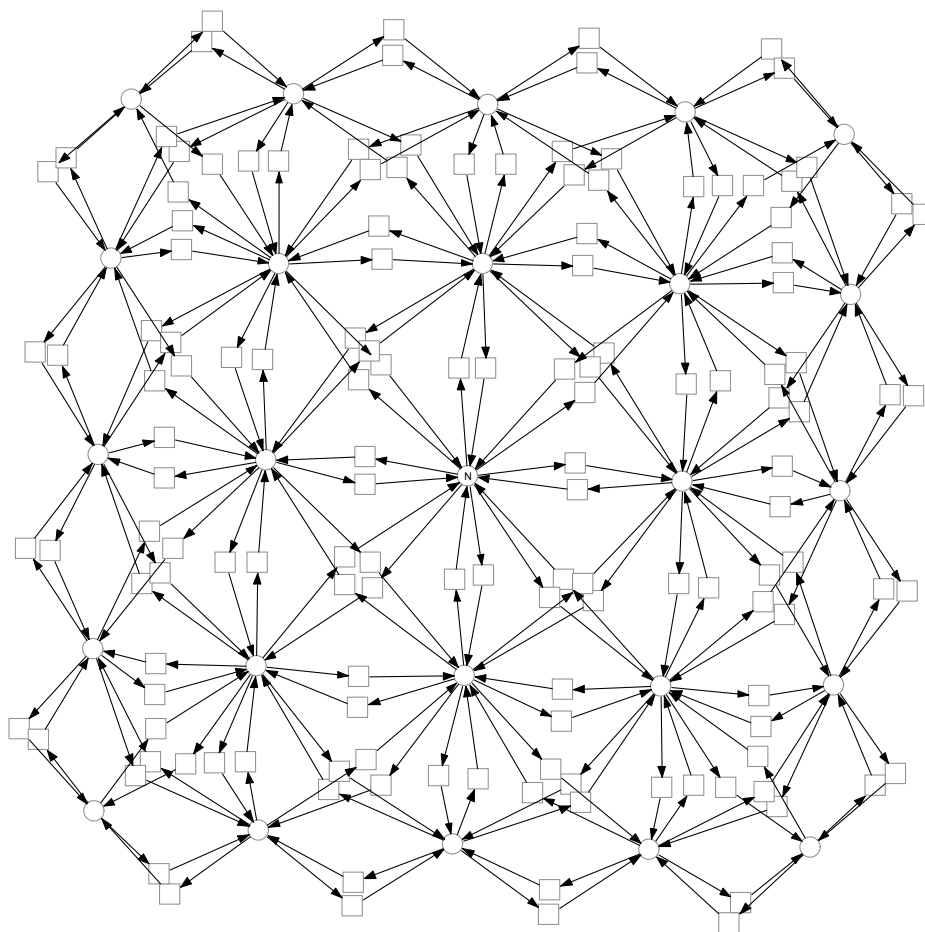


*This form is a summary description of the model entitled “Diffusion2D” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.*

## Description

Diffusion in space is a basic process underlying many spatial (bio-) chemical processes, however typically considered either in the stochastic or continuous setting. The Petri net given here comes from [GHLS13], where it has been used to illustrate the generic modelling of space by use of coloured Petri nets. We discretise the space by a  $D \times D$  rectangular grid,  $D$  being a model parameter, and deploy the 8-neighbourhood relation with reflecting boundary condition. The process starts with  $N$  tokens in the centre position. This model is easily scalable with well-known size of the model growth and its state space.



*Graphical representation for  $D = 5$ , generated by unfolding and automatic layout of a coloured Petri net using Snoopy. There are  $N$  tokens in the centre position. All node names are set to invisible.*

## References

**GHLS13** D. Gilbert, M. Heiner, F. Liu and N. Saunders: Colouring Space - A Coloured Framework for Spatial Modelling in Systems Biology; In Proc. PETRI NETS 2013, Milano, Springer, LNCS, volume 7927, pages 230–249, June 2013.

## Scaling parameter

Parameter name	Parameter description	Chosen parameter values
$D, N$	grid size; i.e. there are $D \times D$ grid positions, and initially $N$ tokens in the centre position.	$\langle D = 5, N = 10 \rangle, \langle D = 5, N = 50 \rangle,$ $\langle D = 5, N = 100 \rangle, \langle D = 5, N = 150 \rangle,$ $\langle D = 5, N = 200 \rangle, \langle D = 5, N = 250 \rangle,$ $\langle D = 5, N = 300 \rangle, \langle D = 5, N = 350 \rangle,$ $\langle D = 10, N = 10 \rangle, \langle D = 10, N = 50 \rangle,$ $\langle D = 10, N = 100 \rangle, \langle D = 10, N = 150 \rangle,$ $\langle D = 10, N = 200 \rangle, \langle D = 20, N = 10 \rangle,$ $\langle D = 20, N = 50 \rangle, \langle D = 20, N = 100 \rangle,$ $\langle D = 20, N = 150 \rangle, \langle D = 30, N = 10 \rangle,$ $\langle D = 30, N = 50 \rangle, \langle D = 30, N = 100 \rangle,$ $\langle D = 30, N = 150 \rangle, \langle D = 40, N = 10 \rangle,$ $\langle D = 40, N = 50 \rangle, \langle D = 40, N = 100 \rangle,$ $\langle D = 40, N = 150 \rangle, \langle D = 50, N = 10 \rangle,$ $\langle D = 50, N = 50 \rangle, \langle D = 50, N = 100 \rangle,$ $\langle D = 50, N = 150 \rangle$

## Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
$D$	$D^2$	$8D^2 - 12D + 4$	$2 \cdot  T $
$\langle D = 5, N = \dots \rangle$	25	144	288
$\langle D = 10, N = \dots \rangle$	100	684	1368
$\langle D = 20, N = \dots \rangle$	400	2964	5928
$\langle D = 30, N = \dots \rangle$	900	6844	13688
$\langle D = 40, N = \dots \rangle$	1600	12324	24648
$\langle D = 50, N = \dots \rangle$	2500	19404	38808

## Structural properties

- ordinary** — all arcs have multiplicity one ..... ✓
- simple free choice** — all transitions sharing a common input place have no other input place ..... ✓ (a)
- extended free choice** — all transitions sharing a common input place have the same input places ..... ✓ (b)
- state machine** — every transition has exactly one input place and exactly one output place ..... ✓ (c)
- marked graph** — every place has exactly one input transition and exactly one output transition ..... ✗ (d)
- connected** — there is an undirected path between every two nodes (places or transitions) ..... ✓ (e)
- strongly connected** — there is a directed path between every two nodes (places or transitions) ..... ✓ (f)
- source place(s)** — one or more places have no input transitions ..... ✗ (g)
- sink place(s)** — one or more places have no output transitions ..... ✗ (h)
- source transition(s)** — one or more transitions have no input places ..... ✗ (i)
- sink transitions(s)** — one or more transitions have no output places ..... ✗ (j)
- loop-free** — no transition has an input place that is also an output place ..... ✓ (k)
- conservative** — for each transition, the number of input arcs equals the number of output arcs ..... ✓ (l)

(a) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (b) stated by [CÆSAR.BDD](#) version 2.6 on all 29 instances (see all aforementioned scaling parameter values).  
 (c) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (d) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (e) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (f) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (g) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (h) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (i) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (j) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (k) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (l) stated by [CÆSAR.BDD](#) version 2.0 on all 29 instances (see all aforementioned scaling parameter values).

**subconservative** — for each transition, the number of input arcs equals or exceeds the number of output arcs ..... ✓<sup>(m)</sup>  
**nested units** — places are structured into hierarchically nested sequential units<sup>(n)</sup> ..... ✗

## Behavioural properties

**safe** — in every reachable marking, there is no more than one token on a place ..... ✗<sup>(o)</sup>  
**deadlock** — there exists a reachable marking from which no transition can be fired ..... ✗  
**reversible** — from every reachable marking, there is a transition path going back to the initial marking ..... ✓  
**quasi-live** — for every transition  $t$ , there exists a reachable marking in which  $t$  can fire ..... ✓<sup>(p)</sup>  
**live** — for every transition  $t$ , from every reachable marking, one can reach a marking in which  $t$  can fire ..... ✓

## Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
$D, N$	$\frac{(D^2+N-1)!}{(D^2-1)! \cdot N!}$	?	$N$	$N$ <sup>(q)</sup>
5, 10	1.3113E+8 <sup>(r)</sup>	5.5537E+9 <sup>(s)</sup>	10 <sup>(t)</sup>	10 <sup>(u)</sup>
5, 50	1.7530E+19 <sup>(v)</sup>	1.7056E+21 <sup>(w)</sup>	50 <sup>(x)</sup>	50 <sup>(y)</sup>
5, 100	2.6011E+25 <sup>(z)</sup>	3.0206E+27 <sup>(aa)</sup>	100 <sup>(ab)</sup>	100 <sup>(ac)</sup>
5, 150	1.8126E+29 <sup>(ad)</sup>	2.2501E+31 <sup>(ae)</sup>	150 <sup>(af)</sup>	150 <sup>(ag)</sup>
5, 200	1.1439E+32 <sup>(ah)</sup>	1.4707E+34 <sup>(ai)</sup>	200 <sup>(aj)</sup>	200 <sup>(ak)</sup>
5, 250	1.8314E+34 <sup>(al)</sup>	2.4062E+36 <sup>(am)</sup>	250 <sup>(an)</sup>	250 <sup>(ao)</sup>
5, 300	1.2054E+36 <sup>(ap)</sup>	1.6072E+38 <sup>(aq)</sup>	300 <sup>(ar)</sup>	300 <sup>(as)</sup>
5, 350	4.2539E+37 <sup>(at)</sup>	?	350 <sup>(au)</sup>	350 <sup>(av)</sup>
10, 10	4.2634E+13 <sup>(aw)</sup>	2.6754E+15 <sup>(ax)</sup>	10 <sup>(ay)</sup>	10 <sup>(az)</sup>
10, 50	1.3419E+40 <sup>(ba)</sup>	3.0801E+42 <sup>(bb)</sup>	50 <sup>(bc)</sup>	50 <sup>(bd)</sup>
10, 100	4.5274E+58 <sup>(be)</sup>	1.5562E+61 <sup>(bf)</sup>	100 <sup>(bg)</sup>	100 <sup>(bh)</sup>
10, 150	2.4252E+71 <sup>(bi)</sup>	?	150 <sup>(bj)</sup>	150 <sup>(bk)</sup>
10, 200	1.3861E+81 <sup>(bl)</sup>	?	200 <sup>(bm)</sup>	200 <sup>(bn)</sup>
20, 10	3.2308E+19 <sup>(bo)</sup>	2.3414E+21 <sup>(bp)</sup>	10 <sup>(bq)</sup>	10 <sup>(br)</sup>
20, 50	7.9113E+66 <sup>(bs)</sup>	2.6113E+69 <sup>(bt)</sup>	50 <sup>(bu)</sup>	50 <sup>(bv)</sup>
20, 100	1.6334E+107 <sup>(bw)</sup>	?	100 <sup>(bx)</sup>	100 <sup>(by)</sup>
20, 150	2.5424E+138 <sup>(bz)</sup>	?	150 <sup>(ca)</sup>	150 <sup>(cb)</sup>
30, 10	1.0100E+23 <sup>(cc)</sup>	7.6041E+24 <sup>(cd)</sup>	10 <sup>(ce)</sup>	10 <sup>(cf)</sup>
30, 50	6.4510E+83 <sup>(cg)</sup>	?	50 <sup>(ch)</sup>	50 <sup>(ci)</sup>
30, 100	5.7466E+139 <sup>(cj)</sup>	?	100 <sup>(ck)</sup>	100 <sup>(cl)</sup>
30, 150	?	?	?	150
40, 10	3.1162E+25 <sup>(cm)</sup>	?	10 <sup>(cn)</sup>	10 <sup>(co)</sup>
40, 50	1.1273E+96 <sup>(cp)</sup>	?	50 <sup>(cq)</sup>	50 <sup>(cr)</sup>
40, 100	?	?	?	100
40, 150	?	?	?	150
50, 10	2.6757E+27 <sup>(cs)</sup>	?	10 <sup>(ct)</sup>	10 <sup>(cu)</sup>
50, 50	?	?	?	50
50, 100	?	?	?	100
50, 150	?	?	?	150

<sup>(m)</sup> stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).

<sup>(n)</sup> the definition of Nested-Unit Petri Nets (NUPN) is available from <http://mcc.lip6.fr/nupn.php>

<sup>(o)</sup> stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).

<sup>(p)</sup> stated by CÆSAR.BDD version 2.0 to be true on 21 instance(s) out of 29, and unknown on the remaining 8 instance(s).

<sup>(q)</sup> number of initial tokens, because the net is conservative.

<sup>(r)</sup> computed at MCC'2014 by Marcie, PNMCC, PNXDD, and Stratagem.

<sup>(s)</sup> computed at MCC'2014 by Marcie.

## Other properties

CPI (Covered by P-Invariants), CTI (Covered by T-Invariants). All places of this Petri net are N-bounded, i.e. all tokens can gather on any one place. The net enjoys some symmetries.

- 
- (t) computed at MCC'2014 by Marcie and PNMC.
  - (u) computed at MCC'2014 by Marcie and PNMC.
  - (v) computed at MCC'2014 by Marcie and PNMC.
  - (w) computed at MCC'2014 by Marcie.
  - (x) computed at MCC'2014 by Marcie and PNMC.
  - (y) computed at MCC'2014 by Marcie and PNMC.
  - (z) computed at MCC'2014 by Marcie and PNMC.
  - (aa) computed at MCC'2014 by Marcie.
  - (ab) computed at MCC'2014 by Marcie and PNMC.
  - (ac) computed at MCC'2014 by Marcie and PNMC.
  - (ad) computed at MCC'2014 by Marcie and PNMC.
  - (ae) computed at MCC'2014 by Marcie.
  - (af) computed at MCC'2014 by Marcie and PNMC.
  - (ag) computed at MCC'2014 by Marcie and PNMC.
  - (ah) computed at MCC'2014 by Marcie and PNMC.
  - (ai) computed at MCC'2014 by Marcie.
  - (aj) computed at MCC'2014 by Marcie and PNMC.
  - (ak) computed at MCC'2014 by Marcie and PNMC.
  - (al) computed at MCC'2014 by Marcie and PNMC.
  - (am) computed at MCC'2014 by Marcie.
  - (an) computed at MCC'2014 by Marcie and PNMC.
  - (ao) computed at MCC'2014 by Marcie and PNMC.
  - (ap) computed at MCC'2014 by Marcie and PNMC.
  - (aq) computed at MCC'2014 by Marcie.
  - (ar) computed at MCC'2014 by Marcie and PNMC.
  - (as) computed at MCC'2014 by Marcie and PNMC.
  - (at) computed at MCC'2014 by PNMC.
  - (au) computed at MCC'2014 by PNMC.
  - (av) computed at MCC'2014 by PNMC.
  - (aw) computed at MCC'2014 by Marcie and PNMC.
  - (ax) computed at MCC'2014 by Marcie.
  - (ay) computed at MCC'2014 by Marcie and PNMC.
  - (az) computed at MCC'2014 by Marcie and PNMC.
  - (ba) computed at MCC'2014 by Marcie and PNMC.
  - (bb) computed at MCC'2014 by Marcie.
  - (bc) computed at MCC'2014 by Marcie and PNMC.
  - (bd) computed at MCC'2014 by Marcie and PNMC.
  - (be) computed at MCC'2014 by Marcie and PNMC.
  - (bf) computed at MCC'2014 by Marcie.
  - (bg) computed at MCC'2014 by Marcie and PNMC.
  - (bh) computed at MCC'2014 by Marcie and PNMC.
  - (bi) computed at MCC'2014 by Marcie and PNMC.
  - (bj) computed at MCC'2014 by Marcie and PNMC.
  - (bk) computed at MCC'2014 by Marcie and PNMC.
  - (bl) computed at MCC'2014 by PNMC.
  - (bm) computed at MCC'2014 by PNMC.
  - (bn) computed at MCC'2014 by PNMC.
  - (bo) computed at MCC'2014 by Marcie, PNMC, and PNXDD.
  - (bp) computed at MCC'2014 by Marcie.
  - (bq) computed at MCC'2014 by Marcie and PNMC.
  - (br) computed at MCC'2014 by Marcie and PNMC.
  - (bs) computed at MCC'2014 by Marcie and PNMC.
  - (bt) computed at MCC'2014 by Marcie.
  - (bu) computed at MCC'2014 by Marcie and PNMC.
  - (bv) computed at MCC'2014 by Marcie and PNMC.
  - (bw) computed at MCC'2014 by PNMC.
  - (bx) computed at MCC'2014 by PNMC.

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- (by) computed at MCC'2014 by PNMIC.
  - (bz) computed at MCC'2014 by PNMIC.
  - (ca) computed at MCC'2014 by PNMIC.
  - (cb) computed at MCC'2014 by PNMIC.
  - (cc) computed at MCC'2014 by Marcie and PNMIC.
  - (cd) computed at MCC'2014 by Marcie.
  - (ce) computed at MCC'2014 by Marcie and PNMIC.
  - (cf) computed at MCC'2014 by Marcie and PNMIC.
  - (cg) computed at MCC'2014 by PNMIC.
  - (ch) computed at MCC'2014 by PNMIC.
  - (ci) computed at MCC'2014 by PNMIC.
  - (ej) computed at MCC'2014 by PNMIC.
  - (ck) computed at MCC'2014 by PNMIC.
  - (cl) computed at MCC'2014 by PNMIC.
  - (cm) computed at MCC'2014 by PNMIC.
  - (cn) computed at MCC'2014 by PNMIC.
  - (co) computed at MCC'2014 by PNMIC.
  - (cp) computed at MCC'2014 by PNMIC.
  - (cq) computed at MCC'2014 by PNMIC.
  - (cr) computed at MCC'2014 by PNMIC.
  - (cs) computed at MCC'2014 by PNMIC.
  - (ct) computed at MCC'2014 by PNMIC.
  - (cu) computed at MCC'2014 by PNMIC.