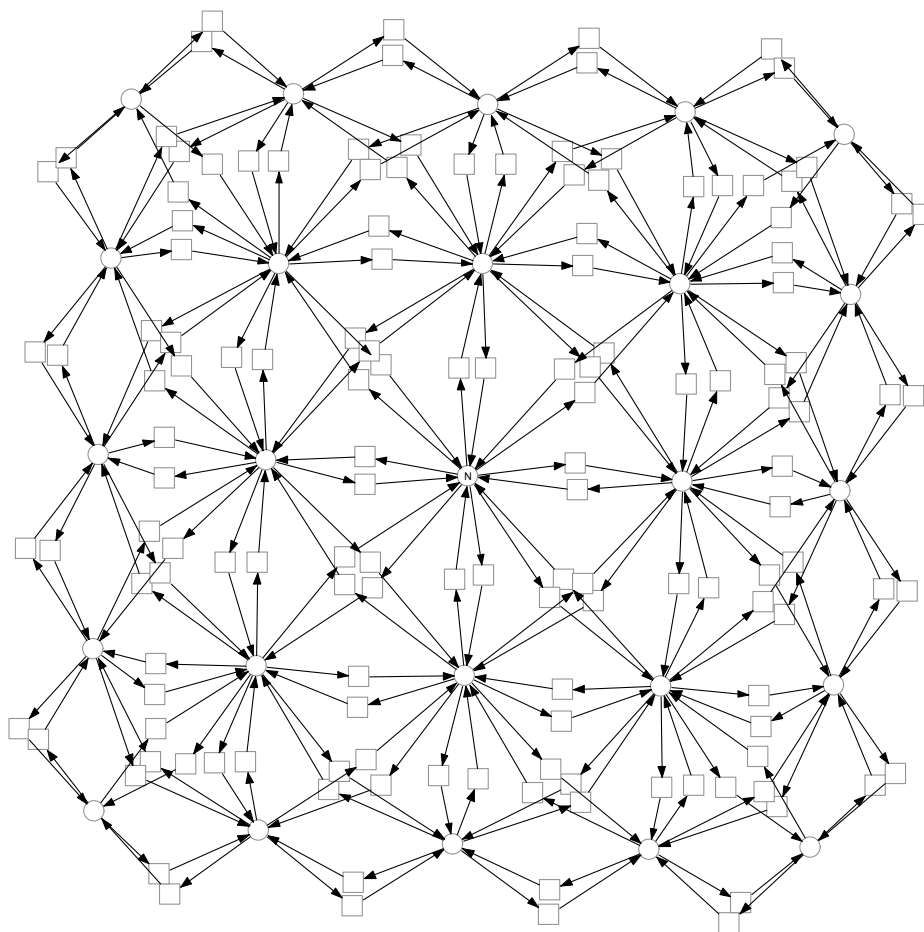


*This form is a summary description of the model entitled “Diffusion2D” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.*

## Description

Diffusion in space is a basic process underlying many spatial (bio-) chemical processes, however typically considered either in the stochastic or continuous setting. The Petri net given here comes from [GHLS13], where it has been used to illustrate the generic modelling of space by use of coloured Petri nets. We discretise the space by a  $D \times D$  rectangular grid,  $D$  being a model parameter, and deploy the 8-neighbourhood relation with reflecting boundary condition. The process starts with  $N$  tokens in the centre position. This model is easily scalable with well-known size of the model growth and its state space.



*Graphical representation for  $D = 5$ , generated by unfolding and automatic layout of a coloured Petri net using Snoopy. There are  $N$  tokens in the centre position. All node names are set to invisible.*

## References

**GHLS13** D. Gilbert, M. Heiner, F. Liu and N. Saunders: Colouring Space - A Coloured Framework for Spatial Modelling in Systems Biology; In Proc. PETRI NETS 2013, Milano, Springer, LNCS, volume 7927, pages 230–249, June 2013.

## Scaling parameter

Parameter name	Parameter description	Chosen parameter values
$D, N$	grid size; i.e. there are $D \times D$ grid positions, and initially $N$ tokens in the centre position.	$\langle D = 5, N = 10 \rangle, \langle D = 5, N = 50 \rangle,$ $\langle D = 5, N = 100 \rangle, \langle D = 5, N = 150 \rangle,$ $\langle D = 5, N = 200 \rangle, \langle D = 5, N = 250 \rangle,$ $\langle D = 5, N = 300 \rangle, \langle D = 5, N = 350 \rangle,$ $\langle D = 10, N = 10 \rangle, \langle D = 10, N = 50 \rangle,$ $\langle D = 10, N = 100 \rangle, \langle D = 10, N = 150 \rangle,$ $\langle D = 10, N = 200 \rangle, \langle D = 20, N = 10 \rangle,$ $\langle D = 20, N = 50 \rangle, \langle D = 20, N = 100 \rangle,$ $\langle D = 20, N = 150 \rangle, \langle D = 30, N = 10 \rangle,$ $\langle D = 30, N = 50 \rangle, \langle D = 30, N = 100 \rangle,$ $\langle D = 30, N = 150 \rangle, \langle D = 40, N = 10 \rangle,$ $\langle D = 40, N = 50 \rangle, \langle D = 40, N = 100 \rangle,$ $\langle D = 40, N = 150 \rangle, \langle D = 50, N = 10 \rangle,$ $\langle D = 50, N = 50 \rangle, \langle D = 50, N = 100 \rangle,$ $\langle D = 50, N = 150 \rangle$

## Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
$D$	$D^2$	$8D^2 - 12D + 4$	$2 \cdot  T $
$\langle D = 5, N = \dots \rangle$	25	144	288
$\langle D = 10, N = \dots \rangle$	100	684	1368
$\langle D = 20, N = \dots \rangle$	400	2964	5928
$\langle D = 30, N = \dots \rangle$	900	6844	13688
$\langle D = 40, N = \dots \rangle$	1600	12324	24648
$\langle D = 50, N = \dots \rangle$	2500	19404	38808

## Structural properties

- ordinary** — all arcs have multiplicity one ..... ✓
- simple free choice** — all (different) transitions with a shared input place have no other input place ..... ✓ (a)
- state machine** — every transition has exactly one input place and exactly one output place ..... ✓ (b)
- marked graph** — every place has exactly one input transition and exactly one output transition ..... ✗ (c)
- connected** — there is an undirected path between every two nodes (places or transitions) ..... ✓ (d)
- strongly connected** — there is a directed path between every two nodes (places or transitions) ..... ✓ (e)
- source place(s)** — one or more places have no input transitions ..... ✗ (f)
- sink place(s)** — one or more places have no output transitions ..... ✗ (g)
- source transition(s)** — one or more transitions have no input places ..... ✗ (h)
- sink transitions(s)** — one or more transitions have no output places ..... ✗ (i)
- loop-free** — no transition has an input place that is also an output place ..... ✓ (j)
- conservative** — for each transition, the number of input arcs equals the number of output arcs ..... ✓ (k)
- subconservative** — for each transition, the number of input arcs equals or exceeds the number of output arcs ..... ✓ (l)

(a) stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (b) stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (c) stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (d) stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (e) stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (f) stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (g) stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (h) stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (i) stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (j) stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (k) stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).  
 (l) stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).

nested units — places are structured into hierarchically nested sequential units <sup>(m)</sup> ..... X

## Behavioural properties

safe — in every reachable marking, there is no more than one token on a place ..... X <sup>(n)</sup>  
 deadlock — there exists a reachable marking from which no transition can be fired ..... X  
 reversible — from every reachable marking, there is a transition path going back to the initial marking ..... ✓  
 quasi-live — for every transition  $t$ , there exists a reachable marking in which  $t$  can fire ..... ✓ <sup>(o)</sup>  
 live — for every transition  $t$ , from every reachable marking, one can reach a marking in which  $t$  can fire ..... ✓

## Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
$D, N$	$\frac{(D^2+N-1)!}{(D^2-1)! \cdot N!}$	?	$N$	$N$
5, 10	1.3113E+8 <sup>(p)</sup>	5.5537E+9 <sup>(q)</sup>	10 <sup>(r)</sup>	10 <sup>(s)</sup>
5, 50	1.7530E+19 <sup>(t)</sup>	1.7056E+21 <sup>(u)</sup>	50 <sup>(v)</sup>	50 <sup>(w)</sup>
5, 100	2.6011E+25 <sup>(x)</sup>	3.0206E+27 <sup>(y)</sup>	100 <sup>(z)</sup>	100 <sup>(aa)</sup>
5, 150	1.8126E+29 <sup>(ab)</sup>	2.2501E+31 <sup>(ac)</sup>	150 <sup>(ad)</sup>	150 <sup>(ae)</sup>
5, 200	1.1439E+32 <sup>(af)</sup>	1.4707E+34 <sup>(ag)</sup>	200 <sup>(ah)</sup>	200 <sup>(ai)</sup>
5, 250	1.8314E+34 <sup>(aj)</sup>	2.4062E+36 <sup>(ak)</sup>	250 <sup>(al)</sup>	250 <sup>(am)</sup>
5, 300	1.2054E+36 <sup>(an)</sup>	1.6072E+38 <sup>(ao)</sup>	300 <sup>(ap)</sup>	300 <sup>(aq)</sup>
5, 350	4.2539E+37 <sup>(ar)</sup>	?	350 <sup>(as)</sup>	350 <sup>(at)</sup>
10, 10	4.2634E+13 <sup>(au)</sup>	2.6754E+15 <sup>(av)</sup>	10 <sup>(aw)</sup>	10 <sup>(ax)</sup>
10, 50	1.3419E+40 <sup>(ay)</sup>	3.0801E+42 <sup>(az)</sup>	50 <sup>(ba)</sup>	50 <sup>(bb)</sup>
10, 100	4.5274E+58 <sup>(bc)</sup>	1.5562E+61 <sup>(bd)</sup>	100 <sup>(be)</sup>	100 <sup>(bf)</sup>
10, 150	2.4252E+71 <sup>(bg)</sup>	?	150 <sup>(bh)</sup>	150 <sup>(bi)</sup>
10, 200	1.3861E+81 <sup>(bj)</sup>	?	200 <sup>(bk)</sup>	200 <sup>(bl)</sup>
20, 10	3.2308E+19 <sup>(bm)</sup>	2.3414E+21 <sup>(bn)</sup>	10 <sup>(bo)</sup>	10 <sup>(bp)</sup>
20, 50	7.9113E+66 <sup>(bq)</sup>	2.6113E+69 <sup>(br)</sup>	50 <sup>(bs)</sup>	50 <sup>(bt)</sup>
20, 100	1.6334E+107 <sup>(bu)</sup>	?	100 <sup>(bv)</sup>	100 <sup>(bw)</sup>
20, 150	2.5424E+138 <sup>(bx)</sup>	?	150 <sup>(by)</sup>	<sup>(bz)</sup>
30, 10	1.0100E+23 <sup>(ca)</sup>	7.6041E+24 <sup>(cb)</sup>	10 <sup>(cc)</sup>	10 <sup>(cd)</sup>
30, 50	6.4510E+83 <sup>(ce)</sup>	?	50 <sup>(cf)</sup>	50 <sup>(cg)</sup>
30, 100	5.7466E+139 <sup>(ch)</sup>	?	100 <sup>(ci)</sup>	100 <sup>(cj)</sup>
30, 150	?	?	?	?
40, 10	3.1162E+25 <sup>(ck)</sup>	?	10 <sup>(cl)</sup>	10 <sup>(cm)</sup>
40, 50	1.1273E+96 <sup>(cn)</sup>	?	50 <sup>(co)</sup>	50 <sup>(cp)</sup>
40, 100	?	?	?	?
40, 150	?	?	?	?
50, 10	2.6757E+27 <sup>(cq)</sup>	?	10 <sup>(cr)</sup>	10 <sup>(cs)</sup>
50, 50	?	?	?	?
50, 100	?	?	?	?
50, 150	?	?	?	?

<sup>(m)</sup> the definition of Nested-Unit Petri Nets (NUPN) is available from <http://mcc.lip6.fr/nupn.php>

<sup>(n)</sup> stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).

<sup>(o)</sup> stated by CÆSAR.BDD version 2.0 to be true on 21 instance(s) out of 29, and unknown on the remaining 8 instance(s).

<sup>(p)</sup> computed at MCC'2014 by Marcie, PNMC, PNXDD, and Stratagem.

<sup>(q)</sup> computed at MCC'2014 by Marcie.

<sup>(r)</sup> computed at MCC'2014 by Marcie and PNMC.

<sup>(s)</sup> computed at MCC'2014 by Marcie and PNMC.

<sup>(t)</sup> computed at MCC'2014 by Marcie and PNMC.

## Other properties

CPI (Covered by P-Invariants), CTI (Covered by T-Invariants). All places of this Petri net are N-bounded, i.e. all tokens can gather on any one place. The net enjoys some symmetries.

- 
- (u) computed at MCC'2014 by Marcie.
  - (v) computed at MCC'2014 by Marcie and PNMC.
  - (w) computed at MCC'2014 by Marcie and PNMC.
  - (x) computed at MCC'2014 by Marcie and PNMC.
  - (y) computed at MCC'2014 by Marcie.
  - (z) computed at MCC'2014 by Marcie and PNMC.
  - (aa) computed at MCC'2014 by Marcie and PNMC.
  - (ab) computed at MCC'2014 by Marcie and PNMC.
  - (ac) computed at MCC'2014 by Marcie.
  - (ad) computed at MCC'2014 by Marcie and PNMC.
  - (ae) computed at MCC'2014 by Marcie and PNMC.
  - (af) computed at MCC'2014 by Marcie and PNMC.
  - (ag) computed at MCC'2014 by Marcie.
  - (ah) computed at MCC'2014 by Marcie and PNMC.
  - (ai) computed at MCC'2014 by Marcie and PNMC.
  - (aj) computed at MCC'2014 by Marcie and PNMC.
  - (ak) computed at MCC'2014 by Marcie.
  - (al) computed at MCC'2014 by Marcie and PNMC.
  - (am) computed at MCC'2014 by Marcie and PNMC.
  - (an) computed at MCC'2014 by Marcie and PNMC.
  - (ao) computed at MCC'2014 by Marcie.
  - (ap) computed at MCC'2014 by Marcie and PNMC.
  - (aq) computed at MCC'2014 by Marcie and PNMC.
  - (ar) computed at MCC'2014 by PNMC.
  - (as) computed at MCC'2014 by PNMC.
  - (at) computed at MCC'2014 by PNMC.
  - (au) computed at MCC'2014 by Marcie and PNMC.
  - (av) computed at MCC'2014 by Marcie.
  - (aw) computed at MCC'2014 by Marcie and PNMC.
  - (ax) computed at MCC'2014 by Marcie and PNMC.
  - (ay) computed at MCC'2014 by Marcie and PNMC.
  - (az) computed at MCC'2014 by Marcie.
  - (ba) computed at MCC'2014 by Marcie and PNMC.
  - (bb) computed at MCC'2014 by Marcie and PNMC.
  - (bc) computed at MCC'2014 by Marcie and PNMC.
  - (bd) computed at MCC'2014 by Marcie.
  - (be) computed at MCC'2014 by Marcie and PNMC.
  - (bf) computed at MCC'2014 by Marcie and PNMC.
  - (bg) computed at MCC'2014 by Marcie and PNMC.
  - (bh) computed at MCC'2014 by Marcie and PNMC.
  - (bi) computed at MCC'2014 by Marcie and PNMC.
  - (bj) computed at MCC'2014 by PNMC.
  - (bk) computed at MCC'2014 by PNMC.
  - (bl) computed at MCC'2014 by PNMC.
  - (bm) computed at MCC'2014 by Marcie, PNMC, and PNXDD.
  - (bn) computed at MCC'2014 by Marcie.
  - (bo) computed at MCC'2014 by Marcie and PNMC.
  - (bp) computed at MCC'2014 by Marcie and PNMC.
  - (bq) computed at MCC'2014 by Marcie and PNMC.
  - (br) computed at MCC'2014 by Marcie.
  - (bs) computed at MCC'2014 by Marcie and PNMC.
  - (bt) computed at MCC'2014 by Marcie and PNMC.
  - (bu) computed at MCC'2014 by PNMC.
  - (bv) computed at MCC'2014 by PNMC.
  - (bw) computed at MCC'2014 by PNMC.
  - (bx) computed at MCC'2014 by PNMC.
  - (by) computed at MCC'2014 by PNMC.

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- (bz) computed at MCC'2014 by PNMIC.
  - (ca) computed at MCC'2014 by Marcie and PNMIC.
  - (cb) computed at MCC'2014 by Marcie.
  - (cc) computed at MCC'2014 by Marcie and PNMIC.
  - (cd) computed at MCC'2014 by Marcie and PNMIC.
  - (ce) computed at MCC'2014 by PNMIC.
  - (cf) computed at MCC'2014 by PNMIC.
  - (cg) computed at MCC'2014 by PNMIC.
  - (ch) computed at MCC'2014 by PNMIC.
  - (ci) computed at MCC'2014 by PNMIC.
  - (cj) computed at MCC'2014 by PNMIC.
  - (ck) computed at MCC'2014 by PNMIC.
  - (cl) computed at MCC'2014 by PNMIC.
  - (cm) computed at MCC'2014 by PNMIC.
  - (cn) computed at MCC'2014 by PNMIC.
  - (co) computed at MCC'2014 by PNMIC.
  - (cp) computed at MCC'2014 by PNMIC.
  - (cq) computed at MCC'2014 by PNMIC.
  - (cr) computed at MCC'2014 by PNMIC.
  - (cs) computed at MCC'2014 by PNMIC.