This form is a summary description of the model entitled “Resource Allocation Model” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.

Description

Let us consider a kind of chessboard, whose dimensions are \(n_R (n_R \geq 1)\) and \(n_C (n_C \geq 2)\), respectively, in which each position has a given capacity (let say \(K \geq 1\)) for holding ants. Let us also consider ant processes which traverse the board, either North-South or South-North directions, always jumping from one position to the following one. For safety reasons each ant, before jumping to the next position, books the position he is going to jump over and also the adjacent one in the west side of the target position. Of course, because of the position capacity constraint, no more than \(K\) ants can stay simultaneously in the same position.

In the set of considered specific models, even columns correspond to North-South ant processes, while odd columns correspond to South-North ant processes. The figure sketches a particular board model for \(n_R=3\), \(n_C=5\) and \(K=1\).

The system can be parametrized in three ways, varying each one of \(n_R\), \(n_C\) and \(K\) (here \(K=1\)). When varying \(n_R\) we will call the model a RAS-R, and when varying \(n_C\) we will call it a RAS-C.

These models belong the family of Resource Allocation Systems, RAS. A RAS is composed of a finite set of processes that share in a competitive way a finite set of resources. In a system there can be resources of several types, and for each type there can be several available copies. In this case, the model belong to the family of the \(S^4 PR\) nets, as described in [TrEz2006].

Places \(p_{\ast\ast}\) correspond to state places, while places \(r_{\ast\ast}\) correspond to resource places. Resource places model the state of the resources shared by the ant processes (in this case the state of a resource is identified as its free capacity). State places model the board position where an ant process is at a given moment.
References


The program used to generate the PNML models can be downloaded from:
https://github.com/fernando/Petri-Net-tools/blob/master/model_generator.c

There is a script to generate all the models for a selected set of parameters, at:

Scaling parameter

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Parameter description</th>
<th>Chosen parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>((nR, nC))</td>
<td>Number of (rows, columns)</td>
<td>(3,2), (3,3), (3,5), (3,10), (3,15), (3,20), (3,50), (3,100), (2,2), (3,2), (5,2), (10,2), (15,2), (20,2), (50,2), (100,2)</td>
</tr>
</tbody>
</table>

Size of the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of places</th>
<th>Number of transitions</th>
<th>Number of arcs</th>
</tr>
</thead>
<tbody>
<tr>
<td>((nR, nC))</td>
<td>(2 \cdot nR \cdot nC)</td>
<td>(nC \cdot (nR + 1))</td>
<td>(4 \cdot nR \cdot nC + 2 \cdot nR \cdot (nC - 1))</td>
</tr>
</tbody>
</table>

Structural properties

- free choice — all (different) transitions with a shared input place have no other input place
- state machine — every transition has exactly one input place and exactly one output place
- marked graph — every place has exactly one input transition and exactly one output transition
- connected — there is a undirected path between every two nodes (places or transitions)
- strongly connected — there is a directed path between every two nodes (places or transitions)
- source place(s) — one or more places have no input transitions
- sink place(s) — one or more places have no output transitions
- source transition(s) — one or more transitions have no input places
- sink transition(s) — one or more transitions have no output places
- loop-free — no transition has an input place that is also an output place
- conservative — for each transition, the number of input arcs equals the number of output arcs
- subconservative — for each transition, the number of input arcs equals or exceeds the number of output arcs

Behavioural properties

- safe — in every reachable marking, there is no more than one token on a place
- deadlock — there exists a reachable marking from which no transition can be fired

\(^{(a)}\) stated by \(\text{CESAR.BDD}\) version 1.7 on all 16 instances (see aforementioned parameter values).
\(^{(b)}\) stated by \(\text{CESAR.BDD}\) version 1.7 on all 16 instances (see aforementioned parameter values).
\(^{(c)}\) stated by \(\text{CESAR.BDD}\) version 1.7 on all 16 instances (see aforementioned parameter values).
\(^{(d)}\) stated by \(\text{CESAR.BDD}\) version 1.7 on all 16 instances (see aforementioned parameter values).
\(^{(e)}\) stated by \(\text{CESAR.BDD}\) version 1.7 on all 16 instances (see aforementioned parameter values).
\(^{(f)}\) stated by \(\text{CESAR.BDD}\) version 1.7 on all 16 instances (see aforementioned parameter values).
\(^{(g)}\) stated by \(\text{CESAR.BDD}\) version 1.7 on all 16 instances (see aforementioned parameter values).
\(^{(h)}\) stated by \(\text{CESAR.BDD}\) version 1.7 on all 16 instances (see aforementioned parameter values).
\(^{(i)}\) stated by \(\text{CESAR.BDD}\) version 1.7 on all 16 instances (see aforementioned parameter values).
\(^{(j)}\) stated by \(\text{CESAR.BDD}\) version 1.7 on all 16 instances (see aforementioned parameter values).
\(^{(k)}\) stated by \(\text{CESAR.BDD}\) version 1.7 on all 16 instances (see aforementioned parameter values).
\(^{(l)}\) stated by \(\text{CESAR.BDD}\) version 1.7 on all 16 instances (see aforementioned parameter values).
\(^{(m)}\) stated by \(\text{CESAR.BDD}\) version 2.0 to be true on 13 instance(s) out of 16, and unknown on the remaining 3 instance(s).
\(^{(n)}\) stated by \(\text{CESAR.BDD}\) version 2.0 to be true on 13 instance(s) out of 16, and unknown on the remaining 3 instance(s).
reversible — from every reachable marking, there is a transition path going back to the initial marking .......................... X (o)

quasi-live — for every transition \( t \), there exists a reachable marking in which \( t \) can fire ................................. ✓ (p)

live — for every transition \( t \), from every reachable marking, one can reach a marking in which \( t \) can fire ............................... X (q)

### Size of the marking graphs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of reachable markings</th>
<th>Number of transition firings</th>
<th>Max. number of tokens per place</th>
<th>Max. number of tokens per marking</th>
</tr>
</thead>
<tbody>
<tr>
<td>((nR, nC) = 2, 2)</td>
<td>8 ((^1))</td>
<td>?</td>
<td>1</td>
<td>(\in [4, 8] ((^x))</td>
</tr>
<tr>
<td>((nR, nC) = 3, 2)</td>
<td>20 ((^1))</td>
<td>?</td>
<td>1</td>
<td>(\in [6, 12] ((^x))</td>
</tr>
<tr>
<td>((nR, nC) = 3, 3)</td>
<td>92 ((^b))</td>
<td>?</td>
<td>1</td>
<td>(\in [9, 18] ((^x))</td>
</tr>
<tr>
<td>((nR, nC) = 3, 5)</td>
<td>1200 ((^y))</td>
<td>?</td>
<td>1</td>
<td>(\in [15, 30] ((^x))</td>
</tr>
<tr>
<td>((nR, nC) = 3, 10)</td>
<td>823552 ((^w))</td>
<td>?</td>
<td>1</td>
<td>(\in [30, 60] ((^x))</td>
</tr>
<tr>
<td>((nR, nC) = 3, 15)</td>
<td>(5.789 \times 10^9 ((^y))</td>
<td>?</td>
<td>1</td>
<td>(\in [45, 90] ((^x))</td>
</tr>
<tr>
<td>((nR, nC) = 3, 20)</td>
<td>(4.065 \times 10^{11} ((^y))</td>
<td>?</td>
<td>1</td>
<td>(\in [60, 120] ((^x))</td>
</tr>
<tr>
<td>((nR, nC) = 3, 50)</td>
<td>(4.872 \times 10^{28} ((^y))</td>
<td>?</td>
<td>1</td>
<td>(\geq 150 ((^a, a))</td>
</tr>
<tr>
<td>((nR, nC) = 3, 100)</td>
<td>(1.420 \times 10^{47} ((^d))</td>
<td>?</td>
<td>1</td>
<td>(\geq 300 ((^x))</td>
</tr>
<tr>
<td>((nR, nC) = 5, 2)</td>
<td>112 ((^ae))</td>
<td>?</td>
<td>1</td>
<td>(\in [10, 20] ((^x))</td>
</tr>
<tr>
<td>((nR, nC) = 10, 2)</td>
<td>6144 ((^ad))</td>
<td>?</td>
<td>1</td>
<td>(\in [20, 40] ((^x))</td>
</tr>
<tr>
<td>((nR, nC) = 15, 2)</td>
<td>278528 ((^ae))</td>
<td>?</td>
<td>1</td>
<td>(\in [30, 60] ((^x))</td>
</tr>
<tr>
<td>((nR, nC) = 20, 2)</td>
<td>(1.153 \times 10^{17} ((^a))</td>
<td>?</td>
<td>1</td>
<td>(\in [40, 80] ((^x))</td>
</tr>
<tr>
<td>((nR, nC) = 50, 2)</td>
<td>(2.927 \times 10^{16} ((^a))</td>
<td>?</td>
<td>1</td>
<td>(\in [100, 200] ((^x))</td>
</tr>
<tr>
<td>((nR, nC) = 100, 2)</td>
<td>(6.465 \times 10^{41} ((^ah))</td>
<td>?</td>
<td>1</td>
<td>(\geq 200 ((^a, a))</td>
</tr>
</tbody>
</table>

Other properties

- For each resource, \( r_{i,j} \), the set \( \{r_{j,i}, p_{j,i}, p_{i,j}(i+1)_j\} \) \((\{r_{i,j}, p_{j,i}\} \text{ for the most eastern process}) is the support of a (minimal) 1-valued P-semiflow, stating the conservativeness of the resource capacity. The whole set of that P-semiflow forms a basis of the set of P-semiflows.

- For each ant process \( j \), the set of involved transitions \( \{t_{j,0}, t_{j,1}, \ldots, t_{j,nR}\} \) is the support of a (minimal) 1-valued T-semiflow, stating the repetitiveness of the process. The whole set of that T-semiflow forms a basis of the set of T-semiflows.

\((o)\) the marking graph has deadlocks and contains more than one reachable marking.

\((p)\) stated by CÆSAR.BDD version 2.0 to be true on 13 instance(s) out of 16, and unknown on the remaining 3 instance(s).

\((x)\) lower and upper bounds given by the number of initial tokens and the number of places.

\((y)\) lower and upper bounds given by the number of initial tokens and the number of places.

\((a)\) lower bound given by the number of initial tokens.

\((b)\) computed by alpina, ITS-Tools, marcie, neco, and pnxdd at MCC’2013; confirmed by CÆSAR.BDD version 1.8.

\((c)\) computed by ITS-Tools, marcie, neco, and pnxdd at MCC’2013; confirmed by CÆSAR.BDD version 1.8.

\((d)\) computed by ITS-Tools, marcie, neco, and pnxdd at MCC’2013; confirmed by CÆSAR.BDD version 1.8.