Description

This is a model of the Peterson’s algorithm for the mutual exclusion problem, in its generalized version for \( N \) processes. This algorithm is based on shared memory communication and uses a loop with \( N-1 \) iterations, each iteration is in charge of stopping one of the competing processes.

Graphical representation for \( N = 2 \)

References

http://dblp.uni-trier.de/rec/bibtex/journals/ipl/Peterson81
Scaling parameter

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Parameter description</th>
<th>Chosen parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td></td>
<td>2, 3, 4, 5, 6, 7</td>
</tr>
</tbody>
</table>

\( N \) is the number of processes. It has an impact on the initial marking of places Idle, Turn and WantSection. It has, also, an impact on the guards of transitions ProgressTurn and Loop. The color functions between EndTurn and AccessCS, as well as the one between IsEndLoop and EndLoop are impacted.

Size of the colored net model

- number of places: 11
- number of transitions: 14
- number of arcs: 42

Size of the derived P/T model instances

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of places</th>
<th>Number of transitions</th>
<th>Number of arcs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 2 )</td>
<td>102</td>
<td>126</td>
<td>384</td>
</tr>
<tr>
<td>( N = 3 )</td>
<td>244</td>
<td>332</td>
<td>1016</td>
</tr>
<tr>
<td>( N = 4 )</td>
<td>480</td>
<td>690</td>
<td>2120</td>
</tr>
<tr>
<td>( N = 5 )</td>
<td>834</td>
<td>1242</td>
<td>3828</td>
</tr>
<tr>
<td>( N = 6 )</td>
<td>1330</td>
<td>2030</td>
<td>6272</td>
</tr>
<tr>
<td>( N = 7 )</td>
<td>1992</td>
<td>3096</td>
<td>9584</td>
</tr>
</tbody>
</table>

Structural properties

- free choice — all (different) transitions with a shared input place have no other input place .......................................................... \( \checkmark \) (a)
- state machine — every transition has exactly one input place and exactly one output place ................................................................. \( \checkmark \) (b)
- marked graph — every place has exactly one input transition and exactly one output transition ........................................................... \( \checkmark \) (c)
- connected — there is an undirected path between every two nodes (places or transitions) .............................................................. \( \checkmark \) (d)
- strongly connected — there is a directed path between every two nodes (places or transitions) ......................................................... \( \checkmark \) (e)
- source place(s) — one or more places have no input transitions ........................................................................................................ \( \times \) (f)
- sink place(s) — one or more places have no output transitions ......................................................................................................... \( \times \) (g)
- source transition(s) — one or more transitions have no input places ................................................................................................. \( \times \) (h)
- sink transitions(s) — one or more transitions have no output places ................................................................................................. \( \times \) (i)
- loop-free — no transition has an input place that is also an output place .......................................................................................... \( \times \) (j)
- conservative — for each transition, the number of input arcs equals the number of output arcs ......................................................... \( \checkmark \) (k)
- subconservative — for each transition, the number of input arcs equals or exceeds the number of output arcs ........................ \( \checkmark \) (l)

(a) stated by CÆSAR.BDD version 1.7 on all 6 instances (2, 3, 4, 5, 6, and 7).
(b) stated by CÆSAR.BDD version 1.7 on all 6 instances (2, 3, 4, 5, 6, and 7).
(c) stated by CÆSAR.BDD version 1.7 on all 6 instances (2, 3, 4, 5, 6, and 7).
(d) stated by CÆSAR.BDD version 1.7 on all 6 instances (2, 3, 4, 5, 6, and 7).
(e) stated by CÆSAR.BDD version 1.7 on all 6 instances (2, 3, 4, 5, 6, and 7).
(f) stated by CÆSAR.BDD version 1.7 on all 6 instances (2, 3, 4, 5, 6, and 7).
(g) stated by CÆSAR.BDD version 1.7 on all 6 instances (2, 3, 4, 5, 6, and 7).
(h) stated by CÆSAR.BDD version 1.7 on all 6 instances (2, 3, 4, 5, 6, and 7).
(i) stated by CÆSAR.BDD version 1.7 on all 6 instances (2, 3, 4, 5, 6, and 7).
(j) stated by CÆSAR.BDD version 1.7 on all 6 instances (2, 3, 4, 5, 6, and 7).
(k) stated by CÆSAR.BDD version 1.7 on all 6 instances (2, 3, 4, 5, 6, and 7).
(l) stated by CÆSAR.BDD version 1.7 on all 6 instances (2, 3, 4, 5, 6, and 7).
Behavioural properties

- **safe** — in every reachable marking, there is no more than one token on a place .......................................................... ✓ (m)
- **deadlock** — there exists a reachable marking from which no transition can be fired ......................................................... X (n)
- **reversible** — from every reachable marking, there is a transition path going back to the initial marking ......................... ✓
- **quasi-live** — for every transition \( t \), there exists a reachable marking in which \( t \) can fire .................................................. ? (o)
- **live** — for every transition \( t \), from every reachable marking, one can reach a marking in which \( t \) can fire ......................... ?

Size of the marking graphs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of reachable markings</th>
<th>Number of transition firings</th>
<th>Max. number of tokens per place</th>
<th>Max. number of tokens per marking</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 2 )</td>
<td>20,754 (^{(p)})</td>
<td>?</td>
<td>1</td>
<td>( \in [8, 102] )^{(q)}</td>
</tr>
<tr>
<td>( N = 3 )</td>
<td>( 3.408 \times 10^{(q)} )</td>
<td>?</td>
<td>?</td>
<td>( \geq 11 )^{(r)}</td>
</tr>
<tr>
<td>( N = 4 )</td>
<td>( 6.299 \times 10^{(r)} )</td>
<td>?</td>
<td>?</td>
<td>( \geq 14 )</td>
</tr>
<tr>
<td>( N = 5 )</td>
<td>( 1.366 \times 10^{(s)} )</td>
<td>?</td>
<td>?</td>
<td>( \geq 17 )</td>
</tr>
<tr>
<td>( N = 6 )</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>( \geq 20 )</td>
</tr>
<tr>
<td>( N = 7 )</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>( \geq 23 )</td>
</tr>
</tbody>
</table>

\(^{(m)}\) stated by CÆSAR.BDD version 2.0 to be true on 1 instance(s) out of 6, and unknown on the remaining 5 instance(s).

\(^{(n)}\) stated by CÆSAR.BDD version 2.0 to be false on 1 instance(s) out of 6, and unknown on the remaining 5 instance(s).

\(^{(o)}\) found to be true by CÆSAR.BDD version 1.9 on instances 2 and 3.

\(^{(p)}\) computed by alpina, ITS-Tools, marcie, neco, and pnxdd at MCC’2013; confirmed by CÆSAR.BDD version 1.8.

\(^{(q)}\) lower and upper bounds given by the number of initial tokens and the number of places.

\(^{(r)}\) computed by alpina, ITS-Tools, marcie, and pnxdd at MCC’2013; confirmed by CÆSAR.BDD version 1.8.

\(^{(s)}\) lower bound given by the number of initial tokens.

\(^{(t)}\) computed by ITS-Tools, and pnxdd at MCC’2013.

\(^{(u)}\) computed by ITS-Tools at MCC’2013.