

This form is a summary description of the model entitled "NeighborGrid" proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.

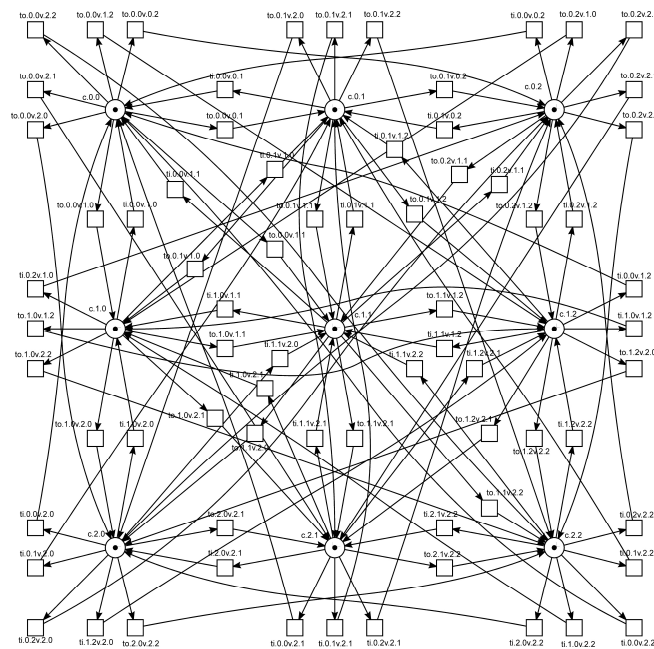
Description

There is a d -dimensional grid of size n indexed with d -tuples having components' range from 0 to $n - 1$. A grid cell model is represented with a single Petri net place denoted as "p". Neighboring cells are connected via pairs of dedicated transitions; transitions are denoted as input "ti" and output "to" with respect to a cell with lesser index. A hypertorus is obtained from a hypercube via closing (connecting) opposite facets in each dimension. Indices are printed with '.' separator on dimensions; character 'v' separates two indices in a couple. More complicated cell models can be inserted but the canvas of connections does not change.

In a generalized neighborhood [1], neighbors are situated at Chebyshev distance equal to 1 restricted by a given interval of Manhattan distance r , $1 \leq r_1 \leq r \leq r_2 \leq d$. Neighbors are connected via facets which are hypercubes having dimensions from $d - r_1$ to $d - r_2$. Thus, $r_1 = 1, r_2 = 1$ gives von-Neumann's neighborhood and $r_1 = 1, r_2 = d$ gives Moore's neighborhood.

A program *hmn* [2] that generates models has the following command line: *hmn d n [m] [e] [r₁] [r₂] > hmn_model.net*

where d is the number of dimensions ($d \geq 1$); n is the size of hypertorus or hypercube ($n \geq 1$; for hypertorus $n \geq 3$), actually the size is $n \times n \times n \times \dots \times n$ (d times); m is the number of tokens in each node ($m \geq 0$, default 1); e is an edge condition: 't' – hypertorus, 'c' – hypercube (default 't'); r_1 is a lower bound of Manhattan distance (default $r_1 = 1$); r_2 is an upper bound of Manhattan distance (default $r_2 = d$), $1 \leq r_1 \leq r_2 \leq d$.



Graphical representation for $d = 2, n = 3, m = 1, e = 't', r_1 = 1, r_2 = 2$

References

- [1] Zaitsev D.A. A generalized neighborhood for cellular automata, Theoretical Computer Science. Online 22 November 2016, <http://dx.doi.org/10.1016/j.tcs.2016.11.002>
- [2] Zaitsev D.A. Generators of canvas for Petri net models of hypertorus (hypercube) grid with Moore's, von-Neumann's, and generalized neighborhoods, <https://github.com/dazeorgacm/hmn>

Scaling parameter

Parameter name	Parameter description	Chosen parameter values
d, n, m, e, r_1, r_2	d – dimension; n – size; m – initial marking of each place; e – edge condition: 't' – hypertorus, 'c' – hypercube; r_1 – lower Manhattan distance; r_2 – upper Manhattan distance	(2,3,1,'t',1,2), (2,3,1,'c',1,2), (3,3,1,'t',1,1), (4,3,2,'c',2,3), (5,4,1,'t',3,5)

Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
$N = (d, n, m, e, r_1, r_2)$	$P = n^d$	for a hypertorus ($e='t'$): $T = n^d \cdot \sum_{j=r_1}^{r_2} 2^j C_d^j$; for a hypercube – sum on all the places (cells) the number of a cell neighbors (depending on the dimension of the corresponding edge)	$A = 2T$
(2,3,3,c,1,2)	9	40	80
(2,3,3,t,1,2)	9	72	144
(3,3,3,t,1,1)	27	162	324
(4,3,3,c,2,3)	81	1632	3264
(5,4,4,t,3,5)	1024	196608	393216

Structural properties

ordinary — all arcs have multiplicity one	✓
simple free choice — all transitions sharing a common input place have no other input place	✓ (a)
extended free choice — all transitions sharing a common input place have the same input places	✓ (b)
state machine — every transition has exactly one input place and exactly one output place	✓ (c)
marked graph — every place has exactly one input transition and exactly one output transition	✗ (d)
connected — there is an undirected path between every two nodes (places or transitions)	✓ (e)
strongly connected — there is a directed path between every two nodes (places or transitions)	✓ (f)
source place(s) — one or more places have no input transitions	✗ (g)
sink place(s) — one or more places have no output transitions	✗ (h)
source transition(s) — one or more transitions have no input places	✗ (i)
sink transitions(s) — one or more transitions have no output places	✗ (j)
loop-free — no transition has an input place that is also an output place	✓ (k)
conservative — for each transition, the number of input arcs equals the number of output arcs	✓ (l)
subconservative — for each transition, the number of input arcs equals or exceeds the number of output arcs	✓ (m)
nested units — places are structured into hierarchically nested sequential units ⁽ⁿ⁾	✗

(a) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).

(b) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).

(c) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).

(d) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).

(e) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).

(f) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).

(g) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).

(h) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).

(i) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).

(j) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).

(k) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).

(l) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).

(m) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).

(n) the definition of Nested-Unit Petri Nets (NUPN) is available from <http://mcc.lip6.fr/nupn.php>

Behavioural properties

safe — <i>in every reachable marking, there is no more than one token on a place</i>	X ^(o)
dead place(s) — <i>one or more places have no token in any reachable marking</i>	X ^(p)
dead transition(s) — <i>one or more transitions cannot fire from any reachable marking</i>	X ^(q)
deadlock — <i>there exists a reachable marking from which no transition can be fired</i>	X
reversible — <i>from every reachable marking, there is a transition path going back to the initial marking</i>	✓ ^(r)
live — <i>for every transition t, from every reachable marking, one can reach a marking in which t can fire</i>	✓ ^(s)

Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
d, n, m, e, r_1, r_2	$C_{(m+1)n^d-1}^{n^d-1}$	Sum on all markings, for all places (cells) with nonzero marking, the number of the corresponding cell neighbors	$m \cdot n^d$	$m \cdot n^d$
(2,3,1,c,1,2)	24310	514800	9	9 ^(t)
(2,3,1,t,1,2)	24310	926640	9	9 ^(u)
(3,3,1,t,1,1)	973469712824056	?	27	27 ^(v)
(4,3,2,c,2,3)	C_{242}^{80}	?	162	162 ^(w)
(5,4,1,t,3,5)	C_{2047}^{1023}	?	1024	1024 ^(x)

^(o) confirmed by [CÆSAR.BDD](#) version 2.7 on all 5 instances (see all aforementioned parameter values).
^(p) stated by [CÆSAR.BDD](#) version 3.3 on all 5 instances (see all aforementioned parameter values).
^(q) stated by [CÆSAR.BDD](#) version 2.7 to be false on 4 instances out of 5, and unknown on the remaining instance.
^(r) stated by [CÆSAR.BDD](#) version 2.7 to be true on 4 instances out of 5, and unknown on the remaining instance.
^(s) stated by [CÆSAR.BDD](#) version 2.7 to be true on 4 instances out of 5, and unknown on the remaining instance.
^(t) number of initial tokens, because the net is conservative.
^(u) number of initial tokens, because the net is conservative.
^(v) number of initial tokens, because the net is conservative.
^(w) number of initial tokens, because the net is conservative.
^(x) number of initial tokens, because the net is conservative.