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This form is a summary description of the model entitled "HypercubeCommunicationGrid" proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.

Description

Hypercube communication grid model [1,2] is composed of nodes which represent data communication equipment (DCE) implementing packet forwarding based on store-and-forward principle. Each DCE has ports, situated on facets of a unit size hypercube, which work in full-duplex mode. Data terminal equipment (DTE) is attached on the hypercube borders. Each DTE receives and sends packets.

Remind that, a d-dimension hypercube has $2 \cdot d$ facets each represents a $(d-1)$-dimension hypercube.

DCE index $(i_1, i_2, ..., i_d)$, where $1 \leq i_j \leq k$, $1 \leq j \leq d$, reflects its location within hypercube. Port index (r, n) consists of dimension number $1 \leq r \leq d$, a facet is perpendicular to, and direction number $1 \leq n \leq 2$, where $n = 1$ represents the direction to the origin of coordinates and $n = 2$ represents the direction to infinity.

DCE model contains an internal buffer represented with $2 \cdot d + 1$ places: the avaliable buffer size and buffer sections for storing packets forwarded to the corresponding ports.

Each of $2 \cdot d$ DCE ports has two tracts: input and output. Memory of a tract is represented with two places – the tract buffer and the tract buffer available capacity (usually equat to unit). An output tract work is modeled by a single transition taking a packet from the corresponding section of the internal buffer and putting it into the tract buffer. An input tract work is modeled by $2 \cdot d - 1$ transitions forwarding arrived packet from the tract buffer to the corresponding section of the internal buffer except of the arrival port number.

A hypercube is composed via meging tract places of neighbor DCE which has a common facet: input tract of one DCE with output tract of the other DCE and vice versa.

On the borders, which constitute $2 \cdot d$ hypercubes of dimension $d - 1$, DTE models are attached. A simple DTE model is represented with a single transition that receives a packet from a neighbor DCE output tract and sends a packet into the neighbor DCE input tract.

For planar case when $d = 2$, models are described in [1,3] with simplified notation of ports.

References

[1] Zaitsev D.A., Zaitsev I.D., Shmeleva T.R. Infinite Petri Nets as Models of Grids (pp. 187-204). Chapter 19 in Mehdi Khosrow-Pour (Ed.) Encyclopedia of Information Science and Technology, Third Edition (10 Volumes). IGI-Global: USA, 2014.

[2] Zaitsev D.A., Shmeleva T.R. Hypercube communication structures analysis via parametric Petri nets. Proceedings of 24th UK Performance Engineering Workshop (UKPEW 2008), 3-4 July 2008, Department of Computing, Imperial College London, p. 358-371.

[3] Shmeleva T.R., Zaitsev D.A., Zaitsev I.D. Analysis of Square Communication Grids via Infinite Petri Nets. Transactions of Odessa National Academy of Telecommunication, no. 1, 2009, p. 27-35.

[4] A C program that generates k^d hypercube can be downloaded from http://daze.ho.ua/tinaz.zip

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Scaling parameter

Parameter name	Parameter description	Chosen parameter values
d, k, p, b	d is the nummber of dimensions; k is the hypercube size of k^d DCE nodes and $2 \cdot d \cdot k^{d-1}$ DTE nodes; p is the number of packets in each section of internal buffer; b is the available size of internal buffer; p and b define the initial marking and do not affect the model structure.	$(d, k) = (3, 4), (4, 3), (5, 3)$ with $p = k$ and $b = d \cdot k$

Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
(d, k)	$P = 6 \cdot d \cdot k^d + k^d + 4 \cdot d \cdot k^{d-1}$	$T = 4 \cdot d^2 \cdot k^d + 2 \cdot d \cdot k^{d-1}$	$A = 16 \cdot d^2 \cdot k^d + 8 \cdot d \cdot k^{d-1}$
$(d = 3, k = 4)$	1408	2400	9600
$(d = 4, k = 3)$	2457	5400	21600
$(d = 5, k = 3)$	9153	25110	100440

Structural properties

ordinary — *all arcs have multiplicity one* .... ✔
simple free choice — *all transitions sharing a common input place have no other input place* .... ✗ (a)
extended free choice — *all transitions sharing a common input place have the same input places* .... ✗ (b)
state machine — *every transition has exactly one input place and exactly one output place* .... ✗ (c)
marked graph — *every place has exactly one input transition and exactly one output transition* .... ✗ (d)
connected — *there is an undirected path between every two nodes (places or transitions)* .... ✔ (e)
strongly connected — *there is a directed path between every two nodes (places or transitions)* .... ✔ (f)
source place(s) — *one or more places have no input transitions* .... ✗ (g)
sink place(s) — *one or more places have no output transitions* .... ✗ (h)
source transition(s) — *one or more transitions have no input places* .... ✗ (i)
sink transitions(s) — *one or more transitions have no output places* .... ✗ (j)
loop-free — *no transition has an input place that is also an output place* .... ✔ (k)
conservative — *for each transition, the number of input arcs equals the number of output arcs* .... ✔ (l)
subconservative — *for each transition, the number of input arcs equals or exceeds the number of output arcs* .... ✔ (m)
nested units — *places are structured into hierarchically nested sequential units* (n) .... ✗

(a) stated by CÆSAR.BDD version 2.2 on all 3 instances $((3, 4), (4, 3), (5, 3))$.
(b) stated by CÆSAR.BDD version 2.6 on all 3 instances $((3, 4), (4, 3), (5, 3))$.
(c) stated by CÆSAR.BDD version 2.2 on all 3 instances $((3, 4), (4, 3), (5, 3))$.
(d) stated by CÆSAR.BDD version 2.2 on all 3 instances $((3, 4), (4, 3), (5, 3))$.
(e) stated by CÆSAR.BDD version 2.2 on all 3 instances $((3, 4), (4, 3), (5, 3))$.
(f) stated by CÆSAR.BDD version 2.2 on all 3 instances $((3, 4), (4, 3), (5, 3))$.
(g) stated by CÆSAR.BDD version 2.2 on all 3 instances $((3, 4), (4, 3), (5, 3))$.
(h) stated by CÆSAR.BDD version 2.2 on all 3 instances $((3, 4), (4, 3), (5, 3))$.
(i) stated by CÆSAR.BDD version 2.2 on all 3 instances $((3, 4), (4, 3), (5, 3))$.
(j) stated by CÆSAR.BDD version 2.2 on all 3 instances $((3, 4), (4, 3), (5, 3))$.
(k) stated by CÆSAR.BDD version 2.2 on all 3 instances $((3, 4), (4, 3), (5, 3))$.
(l) stated by CÆSAR.BDD version 2.2 on all 3 instances $((3, 4), (4, 3), (5, 3))$.
(m) stated by CÆSAR.BDD version 2.2 on all 3 instances $((3, 4), (4, 3), (5, 3))$.
(n) the definition of Nested-Unit Petri Nets (NUPN) is available from `http://mcc.lip6.fr/nupn.php`

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Behavioural properties

safe — *in every reachable marking, there is no more than one token on a place* .... ✗ [(o)]
dead place(s) — *one or more places have no token in any reachable marking* .... ?
dead transition(s) — *one or more transitions cannot fire from any reachable marking* .... ✗
deadlock — *there exists a reachable marking from which no transition can be fired* .... ✗ [(p)]
reversible — *from every reachable marking, there is a transition path going back to the initial marking* .... ✗
live — *for every transition t, from every reachable marking, one can reach a marking in which t can fire* .... ✗

Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
$(d = 3, k = 4)$	?	?	?	2784 [(q)]
$(d = 4, k = 3)$	?	?	?	3780 [(r)]
$(d = 5, k = 3)$	?	?	?	14175 [(s)]

Other properties

Model is $2 \cdot d \cdot p + b$ bounded — the sum of tokens in DCE internal buffer places. Model is P/T-invariant for any natural k as proven in [1,2]

[(o)] stated by CÆSAR.BDD version 2.2 on all 3 instances $((3, 4), (4, 3), (5, 3))$.
[(p)] proven in [1,2]; checked by the Tina `http://www.laas.fr/tina` tool version 3.3.0 as well as other behavioural properties for small values of parameters d, k.
[(q)] number of initial tokens, because the net is conservative.
[(r)] number of initial tokens, because the net is conservative.
[(s)] number of initial tokens, because the net is conservative.