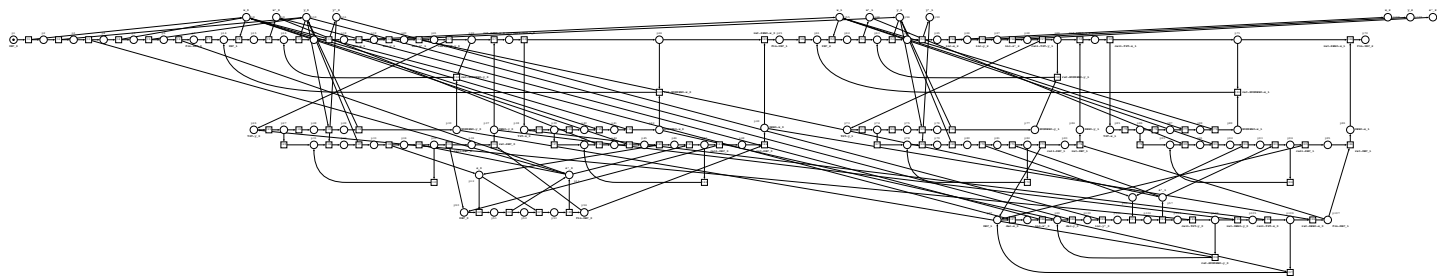


*This form is a summary description of the model entitled “DoubleExponent” proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.*

## Description

The model illustrates difference between weak, strong, and exact Petri net computers [1]. The generated for parameter  $k$  net [2] counts  $2^{2^k}$  as a strong computer after Richard J. Lipton [3]. Well-structured net programs of Javier Esparza [4] have been encoded by Petri nets. The following marking belongs to the reachability space:  $finINC_k = 1$ ;  $x_0 = 2$ ,  $y_0 = 2$ ,  $s'_0 = 2$ ;  $x_1 = 4$ ,  $y_1 = 4$ ,  $s'_1 = 4$ ; ...;  $x_k = 2^{2^k}$ ,  $y_k = 2^{2^k}$ ,  $s'_k = 2^{2^k}$ . It is the only marking with  $finINC_k = 1$ . For a given  $k$ , the resulting net is composed of a single  $Block_0$  and repeated  $(k - 1)$  times  $Block_i$ .  $Block_i$  includes subnets  $INC_{i+1}$ ,  $TSTy_i$ ,  $TSTx_i$ ,  $DEC_i$  and variables (places)  $x_i$ ,  $x'_i$ ,  $y_i$ ,  $y'_i$ ,  $s_i$ ,  $s'_i$ . Connections of neighboring blocks reflect initialization of the next variables  $x_{i+1}$ ,  $y_{i+1}$ ,  $s'_{i+1}$  and recursive calls of the previous block's  $TSTy_{i-1}$ ,  $TSTx_{i-1}$ . Models are generated by program [2] which implements technique described in [5].



Graphical representation for  $k = 2$

## References

- [1] Dmitry Zaitsev, Some Remarks on Petri Net Computers: Weak, Exact, and Strong. Petri Net Newsletter, Volume 85, December 2016, 3-7.
- [2] Dmitry Zaitsev, Generator of Petri nets which count double exponent  $2^{2^k}$  after R.J.Lipton and J.Esparza constructs. <https://github.com/dazeorgacm/depn>
- [3] Richard J. Lipton, The Reachability Problem Requires Exponential Space, Technical Report 63, Yale University, 1976. <http://www.cs.yale.edu/publications/techreports/tr63.pdf>
- [4] Javier Esparza, Decidability and Complexity of Petri Net Problems - An Introduction. LNCS 1491, 1996, 374-428. <http://www7.in.tum.de/um/bibdb/esparza/course.pdf>
- [5] Dmitry A. Zaitsev, Ivan D. Zaitsev and Tatiana R. Shmeleva. Infinite Petri Nets: Part 2, Modeling Triangular, Hexagonal, Hypercube and Hypertorus Structures, Complex Systems, 26(4), 2017, 341-371.

## Scaling parameter

Parameter name	Parameter description	Chosen parameter values
$k$	$k$ – power of double exponent $2^{2^k}$ to compute	1,2,3,4,10,20,100,200

## Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
$k$	$P = 53(k - 1) + 57$	$T = 50(k - 1) + 48$	$A = 141(k - 1) + 135$
$k = 1$	57	48	135
$k = 2$	110	98	276
$k = 3$	163	148	417
$k = 4$	216	198	558
$k = 10$	534	498	1404
$k = 20$	1064	998	2814
$k = 100$	5304	4998	14094
$k = 200$	10604	9998	28194

## Structural properties

<b>ordinary</b> — all arcs have multiplicity one .....	✓
<b>simple free choice</b> — all transitions sharing a common input place have no other input place .....	✗ (a)
<b>extended free choice</b> — all transitions sharing a common input place have the same input places .....	✗ (b)
<b>state machine</b> — every transition has exactly one input place and exactly one output place .....	✗ (c)
<b>marked graph</b> — every place has exactly one input transition and exactly one output transition .....	✗ (d)
<b>connected</b> — there is an undirected path between every two nodes (places or transitions) .....	✓ (e)
<b>strongly connected</b> — there is a directed path between every two nodes (places or transitions) .....	✗ (f)
<b>source place(s)</b> — one or more places have no input transitions .....	✓ (g)
<b>sink place(s)</b> — one or more places have no output transitions .....	✓ (h)
<b>source transition(s)</b> — one or more transitions have no input places .....	✗ (i)
<b>sink transitions(s)</b> — one or more transitions have no output places .....	✗ (j)
<b>loop-free</b> — no transition has an input place that is also an output place .....	✓ (k)
<b>conservative</b> — for each transition, the number of input arcs equals the number of output arcs .....	✗ (l)
<b>subconservative</b> — for each transition, the number of input arcs equals or exceeds the number of output arcs .....	✗ (m)
<b>nested units</b> — places are structured into hierarchically nested sequential units <sup>(n)</sup> .....	✗

## Behavioural properties

<b>safe</b> — in every reachable marking, there is no more than one token on a place .....	✗ (o)
<b>dead place(s)</b> — one or more places have no token in any reachable marking .....	?
<b>dead transition(s)</b> — one or more transitions cannot fire from any reachable marking .....	✗
<b>deadlock</b> — there exists a reachable marking from which no transition can be fired .....	✓
<b>reversible</b> — from every reachable marking, there is a transition path going back to the initial marking .....	✗
<b>live</b> — for every transition $t$ , from every reachable marking, one can reach a marking in which $t$ can fire .....	✗

(a) stated by [CÆSAR.BDD](#) version 2.7 on all 8 instances (for  $k=1, 2, 3, 4, 10, 20, 100$ , and 200).

(b) transitions “t10” and “t23” share a common input place “p10”, but only the former transition has input place “p16”.

(c) stated by [CÆSAR.BDD](#) version 2.7 on all 8 instances (for  $k=1, 2, 3, 4, 10, 20, 100$ , and 200).

(d) stated by [CÆSAR.BDD](#) version 2.7 on all 8 instances (for  $k=1, 2, 3, 4, 10, 20, 100$ , and 200).

(e) stated by [CÆSAR.BDD](#) version 2.7 on all 8 instances (for  $k=1, 2, 3, 4, 10, 20, 100$ , and 200).

(f) from place “p10” one cannot reach place “p1”.

(g) place “p1” is a source place.

(h) stated by [CÆSAR.BDD](#) version 2.7 on all 8 instances (for  $k=1, 2, 3, 4, 10, 20, 100$ , and 200).

(i) stated by [CÆSAR.BDD](#) version 2.7 on all 8 instances (for  $k=1, 2, 3, 4, 10, 20, 100$ , and 200).

(j) stated by [CÆSAR.BDD](#) version 2.7 on all 8 instances (for  $k=1, 2, 3, 4, 10, 20, 100$ , and 200).

(k) stated by [CÆSAR.BDD](#) version 2.7 on all 8 instances (for  $k=1, 2, 3, 4, 10, 20, 100$ , and 200).

(l) stated by [CÆSAR.BDD](#) version 2.7 on all 8 instances (for  $k=1, 2, 3, 4, 10, 20, 100$ , and 200).

(m) stated by [CÆSAR.BDD](#) version 2.7 on all 8 instances (for  $k=1, 2, 3, 4, 10, 20, 100$ , and 200).

(n) the definition of Nested-Unit Petri Nets (NUPN) is available from <http://mcc.lip6.fr/nupn.php>

(o) stated by [CÆSAR.BDD](#) version 2.7 on all 8 instances (for  $k=1, 2, 3, 4, 10, 20, 100$ , and 200).

## Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
$k$	?	?	$2^{2^k}$	$> F, F = 3 \cdot \sum_{i=0}^k 2^{2^i} + 1$ (guess: $F + 2k$ )
1	149	148	4	21
2	3708	3707	16	71
3	2,385,072	2,385,071	256	841
4	$\geq 27^{(p)}$	?	65,536	$\geq 8$
10	$\geq 27^{(q)}$	?	$\approx 1.797693135 \times 10^{308}$	$\geq 8$
20	$\geq 27^{(r)}$	?	$\sim 10^{314.573}$	$\geq 8$
100	?	?	$\sim 10^{38 \cdot 10^{28}}$	$\geq 8$
200	?	?	$\sim 10^{482 \cdot 10^{57}}$	$\geq 8$

## Other properties

The net is a Strong Computer of  $2^{2^k}$  and its size is linear in  $k$  that proves exponential complexity in space of Petri net reachability problem.

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<sup>(p)</sup> stated by [CÆSAR.BDD](#) version 3.3.

<sup>(q)</sup> stated by [CÆSAR.BDD](#) version 3.3.

<sup>(r)</sup> stated by [CÆSAR.BDD](#) version 3.3.