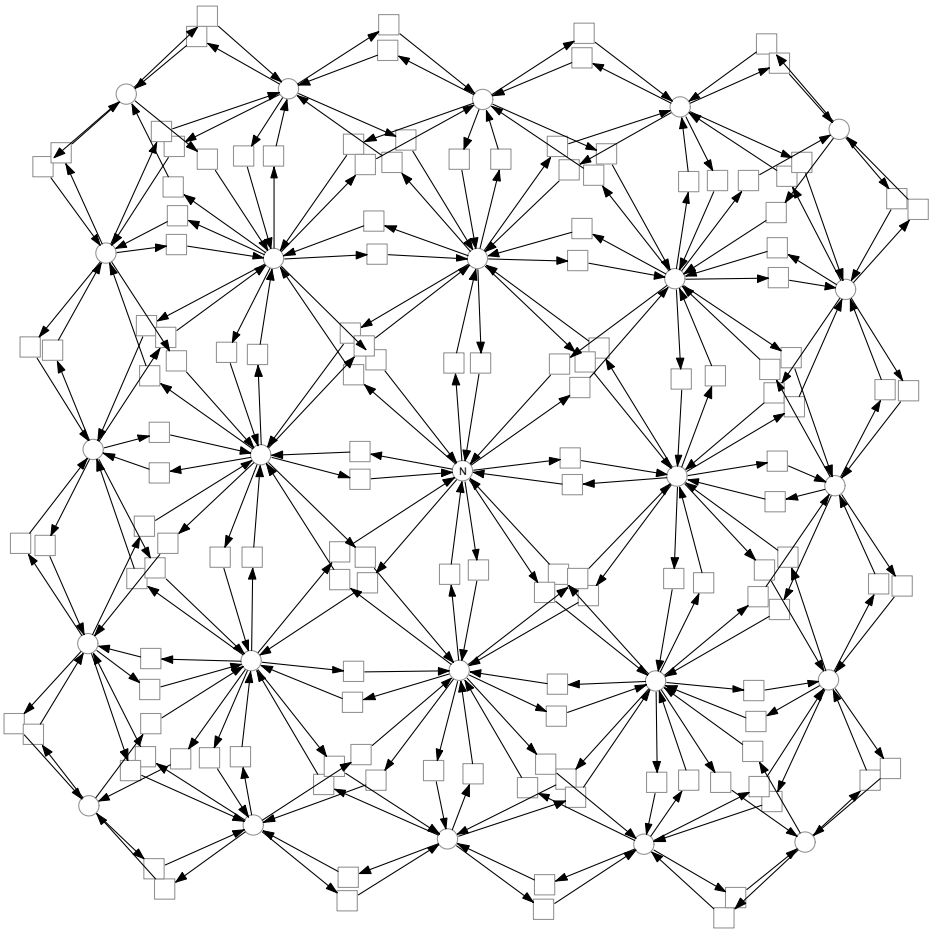
Monika Heiner
monika.heiner@b-tu.de

This form is a summary description of the model entitled "Diffusion2D" proposed for the Model Checking Contest @ Petri Nets. Models can be given in several instances parameterized by scaling parameters. Colored nets can be accompanied by one or many equivalent, unfolded P/T nets. Models are given together with property files (possibly, one per model instance) giving a set of properties to be checked on the model.

Description

Diffusion in space is a basic process underlying many spatial (bio-) chemical processes, however typically considered either in the stochastic or continuous setting. The Petri net given here comes from [GHLS13], where it has been used to illustrate the generic modelling of space by use of coloured Petri nets. We discretise the space by a $D \times D$ rectangular grid, D being a model parameter, and deploy the 8-neighbourhood relation with reflecting boundary condition. The process starts with N tokens in the centre position. This model is easily scalable with well-known size of the model growth and its state space.



Graphical representation for $D = 5$, generated by unfolding and automatic layout of a coloured Petri net using Snoopy. There are N tokens in the centre position. All node names are set to invisible.

References

GHLS13 D. Gilbert, M. Heiner, F. Liu and N. Saunders: Colouring Space - A Coloured Framework for Spatial Modelling in Systems Biology; In Proc. PETRI NETS 2013, Milano, Springer, LNCS, volume 7927, pages 230–249, June 2013.

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Scaling parameter

Parameter name	Parameter description	Chosen parameter values
D, N	grid size; i.e. there are $D \times D$ grid positions, and initially N tokens in the centre position.	$\langle D = 5, N = 10\rangle$, $\langle D = 5, N = 50\rangle$, $\langle D = 5, N = 100\rangle$, $\langle D = 5, N = 150\rangle$, $\langle D = 5, N = 200\rangle$, $\langle D = 5, N = 250\rangle$, $\langle D = 5, N = 300\rangle$, $\langle D = 5, N = 350\rangle$, $\langle D = 10, N = 10\rangle$, $\langle D = 10, N = 50\rangle$, $\langle D = 10, N = 100\rangle$, $\langle D = 10, N = 150\rangle$, $\langle D = 10, N = 200\rangle$, $\langle D = 20, N = 10\rangle$, $\langle D = 20, N = 50\rangle$, $\langle D = 20, N = 100\rangle$, $\langle D = 20, N = 150\rangle$, $\langle D = 30, N = 10\rangle$, $\langle D = 30, N = 50\rangle$, $\langle D = 30, N = 100\rangle$, $\langle D = 30, N = 150\rangle$, $\langle D = 40, N = 10\rangle$, $\langle D = 40, N = 50\rangle$, $\langle D = 40, N = 100\rangle$, $\langle D = 40, N = 150\rangle$, $\langle D = 50, N = 10\rangle$, $\langle D = 50, N = 50\rangle$, $\langle D = 50, N = 100\rangle$, $\langle D = 50, N = 150\rangle$

Size of the model

Parameter	Number of places	Number of transitions	Number of arcs
D	D^2	$8D^2 - 12D + 4$	$2 \cdot \lvert T \rvert$
$\langle D = 5, N = \ldots\rangle$	25	144	288
$\langle D = 10, N = \ldots\rangle$	100	684	1368
$\langle D = 20, N = \ldots\rangle$	400	2964	5928
$\langle D = 30, N = \ldots\rangle$	900	6844	13688
$\langle D = 40, N = \ldots\rangle$	1600	12324	24648
$\langle D = 50, N = \ldots\rangle$	2500	19404	38808

Structural properties

free choice — *all (different) transitions with a shared input place have no other input place* ✔ [a]
state machine — *every transition has exactly one input place and exactly one output place* ✔ [b]
marked graph — *every place has exactly one input transition and exactly one output transition* ✗ [c]
connected — *there is a undirected path between every two nodes (places or transitions)* ✔ [d]
strongly connected — *there is a directed path between every two nodes (places or transitions)* ✔ [e]
source place(s) — *one or more places have no input transitions* ✗ [f]
sink place(s) — *one or more places have no output transitions* ✗ [g]
source transition(s) — *one or more transitions have no input places* ✗ [h]
sink transitions(s) — *one or more transitions have no output places* ✗ [i]
loop-free — *no transition has an input place that is also an output place* ✔ [j]
conservative — *for each transition, the number of input arcs equals the number of output arcs* ✔ [k]
subconservative — *for each transition, the number of input arcs equals or exceeds the number of output arcs* ✔ [l]

[a] stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
[b] stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
[c] stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
[d] stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
[e] stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
[f] stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
[g] stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
[h] stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
[i] stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
[j] stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
[k] stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
[l] stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).

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Behavioural properties

safe — *in every reachable marking, there is no more than one token on a place* ..... ✗ [m]
deadlock — *there exists a reachable marking from which no transition can be fired* ..... ✗
reversible — *from every reachable marking, there is a transition path going back to the initial marking* ..... ✔
quasi-live — *for every transition t, there exists a reachable marking in which t can fire* ..... ✔ [n]
live — *for every transition t, from every reachable marking, one can reach a marking in which t can fire* ..... ✔

Size of the marking graphs

Parameter	Number of reachable markings	Number of transition firings	Max. number of tokens per place	Max. number of tokens per marking
D, N	$(D^2+N-1)!/[(D^2-1)! \cdot N!]$	?	N	N

Other properties

CPI (Covered by P-Invariants), CTI (Covered by T-Invariants). All places of this Petri net are N-bounded, i.e. all tokens can gather on any one place. The net enjoys some symmetries.

[m] stated by CÆSAR.BDD version 2.0 on all 29 instances (see all aforementioned scaling parameter values).
[n] stated by CÆSAR.BDD version 2.0 to be true on 21 instance(s) out of 29, and unknown on the remaining 8 instance(s).